Homework 4

Due: Thursday, April 20

Question 1. A normal population distribution is known to have standard deviation 25. Determine the p-value of a test of the hypothesis that the population mean is equal to 40, if the average of a sample of 49 observations is
(a) 42.5; **(b)** 45.0; **(c)** 47.5.

Solution. $\sigma = 25$, $n = 70$ and $\mu_0 = 40$. Using the formula of two-tailed test

$$p = \mathbb{P}\left(|Z| > \sqrt{n}\left|\frac{\overline{X} - \mu_0}{\sigma}\right|\right) = 2(1 - \mathbb{P}(Z \leq \sqrt{n}\left|\frac{\overline{X} - \mu_0}{\sigma}\right|))$$

(a) $\overline{X} = 42.5$, $p = 2(1 - \mathbb{P}(Z \leq 0.7)) = 0.4839$.
(b) $\overline{X} = 45$, $p = 2(1 - \mathbb{P}(Z \leq 1.4)) = 0.1615$.
(c) $\overline{X} = 47.5$, $p = 2(1 - \mathbb{P}(Z \leq 2.1)) = 0.0357$.

Question 2. The mean breaking strength of a certain type of fiber is required to be at least 190 psi. Past experience indicates that the standard deviation of breaking strength is 5 psi. If a sample of 8 pieces of fiber yielded breakage at the following pressures,

$$189.92, \ 191.72, \ 181.32, \ 200.20, \ 183.03, \ 187.79$$

would you conclude, at the 5 percent level of significance, that the fiber is unacceptable? What about at the 10 percent level of significance?

Solution. Let's say, you are the manufacturer and you are confident about your product, then your null hypothesis is

$$H_0: \ \mu \geq 190$$

So we will conduct a one-tailed test:

$$p = \mathbb{P}\left(Z < \sqrt{n}\left(\frac{\overline{X} - \mu_0}{\sigma}\right)\right) = \mathbb{P}\left(Z < -0.568\right) = 0.285 > 0.1 > 0.05$$

Thus we should accept H_0, and so accept the fiber.
Some may use the H_0 as $\mu \leq 190$, which is valid if you are suspicious about the quality of this fiber. Then the p-value should be 0.715, for which you should also accept the null hypothesis . Note that the choice of null hypothesis depends on the real scenario and it will make a difference in our final conclusion.

Question 3. Twenty years ago, entering male high school students of Central High could do an average of 20 pushups in 60 seconds. To see whether this remains true today, a random sample of

29 freshmen was chosen. If their average was 22.5 with a sample standard deviation of 4.5, can we conclude that the mean is no longer equal to 20? Use the 5 percent level of significance.

Solution. This is a two-tailed test and the null hypothesis is

$$H_0: \ \mu = 20$$

So we calculate the p-value as ($n = 29$, $s = 4.5$, $\overline{X} = 22.5$)

$$p = \mathbb{P}\left(|t| > \sqrt{n}\left|\frac{\overline{X} - \mu_0}{s}\right|\right) = 2(1 - \mathbb{P}(t \leq \sqrt{n}\left|\frac{\overline{X} - \mu_0}{s}\right|)) = 2(1 - P(t_{28} \leq 2.992)) = 0.0057 \leq 0.05$$

Note that we need to check the t-table with $(n-1) = 28$ degrees of freedom. So we should reject the null hypothesis and we can conclude that the mean is no longer equal to 20.

Another method is to compare 2.992 with $t_{28}(0.975) = 2.04$. Since $2.992 > 2.04$, we should reject the null hypothesis at the significance level of 0.05.

Question 4. Last year it was found that on average it took graduate students 20mins to fill out the forms required for graduation. This year the department has changed the form and asked graduating student to report how much time it took them to complete the forms. Of the students 22 replied with their time, the average time that they reported was 18.5 mins, and the sample standard deviation was 5.2. Can we conclude that the new forms take less time to complete then the older forms? Use a 10 percent significance level.
Solution. This is a one-tailed test and the null hypothesis is:

$$H_0 : \mu \geq 20 \tag{1}$$

If we reject this null then we can conclude the new forms take less time. So we can calculate the p-value as ($n = 22, s = 5.2, \bar{X} = 18.5$)

$$p = \mathbb{P}\left(t < \sqrt{n}\left(\frac{\bar{X} - \mu_0}{s}\right)\right) = \mathbb{P}(t_{21} \leq -1.3530) = 0.0952 \leq 0.1 \tag{2}$$

Note that we need to check the t-table with $(n-1) = 21$ degrees of freedom. We should therefore reject the null hypothesis. Alternatively we can test the hypothesis:

$$H_0 : \mu \leq 20 \tag{3}$$

Then we would obtain the p-value 0.9048 for which we would accept H_0.