Department of Industrial Engineering & Operations Research

IEOR 165 (Spring 2017)

Homework 3

Due: Thursday ~~Mar 16~~ April 6

Question 1. Recall from the previous homework the maximum likelihood estimate derived from iid data $X_1, \ldots, X_n$ with the pdf

$$f(x) = \theta^2 x e^{-\theta x}, \quad 0 < x,\ 0 < \theta < \infty$$

Find the bias, variance, and mean squared error (MSE) of the estimate $\hat{\theta}_{mle}$.
Hint: You can use the fact that $X_i \sim \text{Erlang}(2, \theta)$ and that $Y \sim \text{Erlang}(2n, \theta)$ if $Y = \sum_{i=1}^{n} X_i$.

Question 2. In the same setting as problem 1, recall that a shrinkage estimator is an estimator which is a constant multiple of the MLE estimator (i.e. $\hat{\theta}_{shrink} = k\hat{\theta}_{mle}$ for some constant k). Compute an unbiased shrinkage estimator and minimum MSE shrinkage estimator for θ. What is the bias, variance, and MSE of these estimators?

Question 3. River floods are often measured by their discharges (in units of feet cubed per second). The following table gives the flood discharges of the Blackstone River in Woonsocket, Rhode Island, in each of the years from 1956 to 1965.

Year	Flood Discharge (ft^3/s)
1956	6035
1957	6804
1958	5018
1959	5862
1960	8064
1961	8095
1962	4326
1963	4703
1964	3849
1965	5576

Take the yearly flood discharge as a random variable X. Use the above data to estimate the cdf $F_X(5000)$ using empirical distribution function.

Question 4. In the context of Question 3, estimate the density at 5000, i.e. $f_X(5000)$, using histogram method with bin edges $\{0, 2000, 3000, 4000, 4500, 5000, 5500, 6000, 10000\}$.

Question 5. In the context of Question 3, estimate the density at 5000 using kernel density approach. Suppose we choose the uniform kernel and $h = 300$.