**Department of Industrial Engineering & Operations Research**

**IEOR 165 (Spring 2017)**

# Homework 2

**Due: Thursday, March 2**

**Question 1.** Let $X_1, \ldots, X_n$ be iid from the pdf

$$f(x) = \theta^2 x e^{-\theta x}, \quad 0 < x,\ 0 < \theta < \infty$$

Find the MLE of $\theta$.

**Question 2.** Let $X_1, \ldots, X_n$ be iid with pmf

$$f_\theta(x) = \binom{N}{x} \theta^x (1-\theta)^{N-x}, \quad 0 \le x \le N, \quad 0 \le \theta \le 1$$

Where $N$ is a known constant. Find the MLE of $\theta$.

**Question 3.** The chlorine residual (C) in a swimming pool at various times after being cleaned (T) is as given:

| Time (hr) | Chlorine Residual (pt/million) |
|---|---|
| 2 | 1.92 |
| 4 | 1.55 |
| 6 | 1.47 |
| 8 | 1.33 |
| 10 | 1.43 |
| 12 | 1.08 |

Assume the following relationship

$$C \approx a \exp(-bT)$$

What would you predict for the chlorine residual 15 hours after a cleaning?

**Question 4.** Assume we have one observation $X$ drawn from a Bernoulli distribution with unknown parameter $p$. $p$ itself follows a beta distribution with shape parameters $\alpha$ and $\beta$. Show that the posterior distribution is beta and find its mean and variance ($\mathbb{E}[p|X]$ and $Var(p|X)$)
Hint : The p.d.f. of a beat distribution with parameters $\alpha, \beta$ is:

$$f(y) = \frac{y^{\alpha-1}(1-y)^{\beta-1}}{\mathbf{B}(\alpha, \beta)} \tag{1}$$

Where $\mathbf{B}(a, b) = \frac{(a-1)!(b-1)!}{(a+b-1)!}$ is the beta function. The mean is $\frac{\alpha}{\alpha+\beta}$ and the variance is $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

**Question 5.** Find the Maximum *a posteriori* estimate (MAP) of $p$ in Question 4.