Department of Industrial Engineering & Operations Research

IEOR 165 (Spring 2016)

Practice Questions (2nd Half)

Question 1. Suppose on any given day you are either happy, so-so or sad. Assume that the chance of being in these three states tomorrow only depends on the state today. Then your mood transition process can be modeled as a discrete time Markov chain. The following data is available.

	Happy tomorrow	So-so tomorrow	Sad tomorrow
Happy today	120	30	40
So-so today	80	40	50
Sad today	10	60	100

Given the data, estimate the transition probabilities by MLE.

Solution. Denote Happy, So-so and Sad by state 1, 2 and 3. The transition probabilities can be estimated by

$$\hat{P}_{11} = \frac{120}{120 + 30 + 40} = \frac{12}{19}$$

$$\hat{P}_{12} = \frac{30}{120 + 30 + 40} = \frac{3}{19}$$

$$\hat{P}_{13} = \frac{40}{120 + 30 + 40} = \frac{4}{19}$$

$$\hat{P}_{21} = \frac{80}{80 + 40 + 50} = \frac{8}{17}$$

$$\hat{P}_{22} = \frac{30}{80 + 40 + 50} = \frac{4}{17}$$

$$\hat{P}_{23} = \frac{40}{80 + 40 + 50} = \frac{5}{17}$$

$$\hat{P}_{31} = \frac{10}{10 + 60 + 100} = \frac{1}{17}$$

$$\hat{P}_{32} = \frac{60}{10 + 60 + 100} = \frac{6}{17}$$

$$\hat{P}_{33} = \frac{100}{10 + 60 + 100} = \frac{10}{17}$$

Question 2. The mean response time of a species of pigs to a stimulus is 0.8 seconds. 28 pigs were given 2 oz of alcohol and then tested. If their average response time was 1.0 seconds with a standard deviation of 0.3 seconds, can we conclude that alcohol affects the mean response time? Use the 5 percent level of significance and assume there are two-sided deviations.

Solution. The null hypothesis is $\mu = 0.8$. Since only sample standard deviation is known, we should perform a two-sided t-test. The p-value is

$$p = \mathbb{P}\Big(|t| > \left|\frac{\overline{X} - \mu_0}{s/\sqrt{n}}\right|\Big) = 2\mathbb{P}\Big(t_{27} > \left|\frac{1 - 0.8}{0.3/\sqrt{28}}\right|\Big) = 2(1 - \mathbb{P}(t_{27} < 3.528)) = 0.00152$$

So we should reject the null hypothesis and conclude that alcohol affects the mean response time.

Or you can compare 3.528 with $t_{27}(1 - 0.05/2) = 2.052$, since 3.528 > 2.052, we should reject the null hypothesis.

Question 3. The viscosity of two different brands of car oil is measured and the following data resulted:

Brand 1	10.62	10.58	10.33	10.72	10.44	10.74	
Brand 2	10.50	10.52	10.58	10.62	10.55	10.51	10.53

Test the hypothesis that the mean viscosity of the two brands is equal, assuming that the populations have normal distributions with equal variances.

Solution. The null hypothesis is $\mu_1 = \mu_2$. Since the population variance is unknown, we should perform a two-sample unpaired t-test. The *p*-value is

$$p = \mathbb{P}\Big(|t_{n_1+n_2-2}| > \left|\frac{\overline{X}_1 - \overline{X}_2}{s\sqrt{1/n_1 + 1/n_2}}\right|\Big)$$

where

$$s^{2} = \frac{1}{n_{1} + n_{2} - 2} \left(\sum_{i=1}^{n_{1}} (X_{1}^{i} - \overline{X})^{2} + \sum_{i=1}^{n_{1}} (X_{2}^{i} - \overline{X})^{2} \right) = 0.0127$$

 So

$$p = \mathbb{P}\left(|t_{11}| > \left|\frac{0.0274}{\sqrt{0.0127}\sqrt{1/6 + 1/7}}\right|\right) = 2(1 - \mathbb{P}(t_{11} < 0.437)) = 0.671 > 0.05$$

Hence we accept the null hypothesis at the significance level of 0.05 (actually you should accept it at any reasonable significance level).

Question 4. A set of 10 determinations, by a method devised by the chemist Karl Fischer, of the percentage of water in a methanol solution yielded the following data.

0.50, 0.55, 0.53, 0.56, 0.540.57, 0.52, 0.60, 0.55, 0.58 Assuming normality, use these data to give a 95 percent confidence interval for the actual percentage.

Solution. The confidence interval can be derived directly as

$$\underline{\mu} = \overline{X} - s \cdot t_{n-1}(1 - \alpha/2) / \sqrt{n} = 0.55 - 0.0294 \cdot 2.262 / \sqrt{9} = 0.528$$
$$\overline{\mu} = \overline{X} + s \cdot t_{n-1}(1 - \alpha/2) / \sqrt{n} = 0.55 + 0.0294 \cdot 2.262 / \sqrt{9} = 0.572$$

Question 5. The following data present the number of defective products in subgroups of size 100. Under normal conditions, the fraction defective is 0.01.

Subgroup	Number of defective products
1	2
2	1
3	4
4	3
5	2
6	5

Derive the LCL and UCL to control the fraction defective of this process and determine whether the process is in control or not at the significance level of 0.05.

Solution. We know p = 0.01. So the LCL and UCL can be computed as

$$LCL = p - z(1 - \alpha/2) \cdot \sqrt{\frac{p(1 - p)}{n}} = 0.01 - 1.96\sqrt{\frac{0.01 \times 0.99}{100}} = -0.00950$$
$$UCL = p - z(1 - \alpha/2) \cdot \sqrt{\frac{p(1 - p)}{n}} = 0.01 + 1.96\sqrt{\frac{0.01 \times 0.99}{100}} = 0.0295$$

So this process is out of control as the fraction defective of group 3, 4 and 5 all exceed the UCL.