## Department of Industrial Engineering & Operations Research

## **IEOR 165 (Spring 2016)**

## Solution to Practice Questions

**Question 1.** The weekly counts of earthquakes in North America for 8 consecutive weeks (suppose they are iid) are summarized in the following table:

Week	1	2	3	4	5	6	7	8
Counts	7	6	10	9	15	12	14	11

Assuming that the earthquake count follows Poisson distribution with rate  $\lambda$ . Estimate  $\lambda$  using the method of moments.

Solution. Take the first moment as

 $\mu_1 = \lambda$ 

So 
$$\hat{\lambda} = \hat{\mu}_1 = \sum_{i=1}^8 C_i / 8 = 10.5$$

**Question 2.** Suppose you are given the following data set that includes the annual sales in different markets of different size. Construct a linear model to predict the sales in a new market with known size. Please specify the response and the predictor and estimate the corresponding parameters.

Market size (thousand)	20	35	53	59	72
Sales (thousand)	5	7	9	10	11

Solution. The response is Sales and the predictor is Market size. The linear model is

$$Sales = b + m \cdot Market size + \epsilon$$

By OLS, we have

$$\hat{m} = 0.117$$
  $\hat{b} = 2.795$ 

**Question 3.** In the context of Question 2, if we assume  $Var(S|M) \propto M$  (S: Sales, M: Market size), re-estimate the parameters of the linear model you derived in Question 2.

**Solution.** Let weight be  $w_i = \frac{1}{E(\epsilon_i^2|M_i)} = \frac{1}{Var(S_i|M_i)} = \frac{1}{M}$ , then by using weighted least squares, we have

$$\hat{m} = 0.175 \quad b = 0.523$$

**Question 4.** Let  $X_1, \ldots, X_n$  be iid with one of two pdfs. If  $\theta = 0$  then

$$f(x|\theta) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & o.w. \end{cases}$$

while if  $\theta = 1$ 

$$f(x|\theta) = \begin{cases} \frac{1}{2\sqrt{x}} & \text{if } 0 < x < 1\\ 0 & o.w. \end{cases}$$

Find the MLE of  $\theta$ .

**Solution.** The likelihood function is (assume  $0 < X_i < 1$ , otherwise the likelihood function is 0)

$$L(\theta) = \begin{cases} 1 & \theta = 0\\ \prod_{i=1}^{n} \frac{1}{2\sqrt{X_i}} & \theta = 1 \end{cases}$$

So we have

$$\hat{\theta}_{MLE} = \begin{cases} 0 & \text{if } \prod_{i=1}^{n} \frac{1}{2\sqrt{X_i}} \le 1\\ 1 & o.w. \end{cases}$$

Question 5. In the context of Question 1, suppose we do not make any assumption about the distribution of earthquake count, please estimate the density of earthquake count at the point u = 8 by choosing one method you learned in the class. Specify your method and corresponding details.

**Solution.** You can use histograms or kernel method. Here I use kernel method with uniform kernel and h = 2. We first compute  $K(\frac{u-C_i}{h})$  as follows

$$K((7-8)/2) = 0.5$$
  

$$K((6-8)/2) = 0.5$$
  

$$K((10-8)/2) = 0.5$$
  

$$K((9-8)/2) = 0.5$$
  

$$K((15-8)/2) = 0$$
  

$$K((12-8)/2) = 0$$
  

$$K((14-8)/2) = 0$$
  

$$K((11-8)/2) = 0$$

So the kernel density estimate is

$$\hat{f}(u=8) = \frac{1}{nh} \sum_{i=1}^{n} K(\frac{u-C_i}{h}) = \frac{1}{8 \cdot 2} (0.5 + 0.5 + 0.5 + 0.5) = 0.125$$