IEOR165 Discussion
Week 11

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Outline

Revisit One-Sample Test

- How to choose the formula?
- An equivalent method:

$$
\begin{aligned}
& p = P\left(|Z| > \sqrt{n}\cdot\left|\frac{\overline{X}-\mu_0}{\sigma}\right|\right) \geq \alpha \\
\Leftrightarrow\quad & 2P\left(Z > \sqrt{n}\cdot\left|\frac{\overline{X}-\mu_0}{\sigma}\right|\right) \geq \alpha \\
\Leftrightarrow\quad & 2\left[1 - P\left(Z \leq \sqrt{n}\cdot\left|\frac{\overline{X}-\mu_0}{\sigma}\right|\right)\right] \geq \alpha \\
\Leftrightarrow\quad & P\left(Z \leq \sqrt{n}\cdot\left|\frac{\overline{X}-\mu_0}{\sigma}\right|\right) \leq 1-\alpha/2 \\
\Leftrightarrow\quad & \sqrt{n}\cdot\left|\frac{\overline{X}-\mu_0}{\sigma}\right| \leq z(1-\alpha/2)
\end{aligned}
$$

Some common used values: $z(1-0.05/2) = z(0.975) = 1.96$, $z(1-0.1/2) = z(0.95) = 1.645$.

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Example (Chemical pH Testing): $\mu_0 = 8.15$, $\sigma = 0.02$, $\overline{X} = 8.26$ and $n = 5$. Then

$$\sqrt{n} \cdot \left| \frac{\overline{X} - \mu_0}{\sigma} \right| = \sqrt{5} \left| \frac{8.26 - 8.15}{0.02} \right| = 5.5 > 1.96$$

So we reject the null hypothesis with the significance level of 0.05.

When $\overline{X} = \mu_0$, what is your decision?

With unknown variance, we can make the following comparison

$$\sqrt{n} \cdot \left| \frac{\overline{X} - \mu_0}{s} \right| \leq t_{n-1}(1 - \alpha/2) \quad \text{(Accept } H_0\text{)}$$

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Example (Chemical pH Testing): $\mu_0 = 8.15$, $\sigma = 0.02$, $\overline{X} = 8.26$ and $n = 5$. Then

$$\sqrt{n} \cdot \left| \frac{\overline{X} - \mu_0}{\sigma} \right| = \sqrt{5} \left| \frac{8.26 - 8.15}{0.02} \right| = 12.3 > 1.96$$

So we reject the null hypothesis with the significance level of 0.05.

When $\overline{X} = \mu_0$, what is your decision?

With unknown variance, we can make the following comparison

$$\sqrt{n} \cdot \left| \frac{\overline{X} - \mu_0}{s} \right| \leq t_{n-1}(1 - \alpha/2) \quad \text{(Accept } H_0)$$

Revisit One-Sample Test

For the one-tailed test

$$H_0 : \mu \leq \mu_0 \quad H_1 : \mu > \mu_0$$

We have similar result:

$$\begin{aligned} & p = P\Big(Z > \sqrt{n} \cdot (\frac{\overline{X} - \mu_0}{\sigma})\Big) \geq \alpha \\ \Leftrightarrow \quad & 1 - P\Big(Z \leq \sqrt{n} \cdot (\frac{\overline{X} - \mu_0}{\sigma})\Big) \geq \alpha \\ \Leftrightarrow \quad & P\Big(Z \leq \sqrt{n} \cdot (\frac{\overline{X} - \mu_0}{\sigma})\Big) \leq 1 - \alpha \\ \Leftrightarrow \quad & \sqrt{n} \cdot (\frac{\overline{X} - \mu_0}{\sigma}) \leq z(1 - \alpha) \quad (\text{Accept } H_0) \end{aligned}$$

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Example (8.3e)

$$H_0 : \mu = 8 \quad H_1 : \mu > 8$$

The variance is 4 and we get $\overline{X} = 9.5$ with sample size of 5. What can you conclude? (you can get rid of the Z-table) What if $\overline{X} = 7.5$?

Derive the decision rule for

$$H_0 : \mu \geq \mu_0 \quad H_1 : \mu < \mu_0$$

From One-Sample to Two-Sample

Unpaired Case:

$$H_0 : \mu = \mu_0 \quad H_1 : \mu \neq \mu_0$$

$$\Downarrow$$

$$H_0 : \mu_X - \mu_Y = 0 \quad H_1 : \mu_X - \mu_Y \neq 0$$

With known variance:

$$\frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$$

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\sigma_x^2 / n_x + \sigma_y^2 / n_y}} \sim N(0, 1)$$

With unknown variance (equal):

$$\frac{\overline{X} - \mu_0}{s / \sqrt{n}} \sim t_{n-1}$$

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{s^2 (1/n_x + 1/n_y)}} \sim t(0, n + m - 2)$$

From One-Sample to Two-Sample

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EXAMPLE 8.4a Two new methods for producing a tire have been proposed. To ascertain which is superior, a tire manufacturer produces a sample of 10 tires using the first method and a sample of 8 using the second. The first set is to be road tested at location A and the second at location B. It is known from past experience that the lifetime of a tire that is road tested at one of these locations is normally distributed with a mean life due to the tire but with a variance due (for the most part) to the location. Specifically, it is known that the lifetimes of tires tested at location A are normal with standard deviation equal to 4,000 kilometers, whereas those tested at location B are normal with $\sigma = 6{,}000$ kilometers. If the manufacturer is interested in testing the hypothesis that there is no appreciable difference in the mean life of tires produced by either method, what conclusion should be drawn at the 5 percent level of significance if the resulting data are as given in Table 8.3?

TABLE 8.3 *Tire Lives in Units of 100 Kilometers*

Tires Tested at A	Tires Tested at B
61.1	62.2
58.2	56.6
62.3	66.4
64	56.2
59.7	57.4
66.2	58.4
57.8	57.6
61.4	65.4
62.2	
63.6	

Solution

Basics

- Point estimate vs. Interval estimate
  - Point estimate: a single value (our best guess)
  - Interval estimate: an interval that we believe the true value lies within (How confident?)
- Confidence interval is a frequentist method (credible interval for Bayesian)
- The meaning of confidence
  - We are 99% confident that the true value of the parameter is in our confidence interval
  - If we repeat the experiment many times, the calculated confidence interval will include the true value 99% of the time
- Confidence level and significance level: when performing hypothesis testing, a 95% confidence interval reflects a 5% significance level
- When the confidence level encompasses μ_0, we accept the null hypothesis; otherwise we reject it.

Example

Example (8.3e)

$$H_0 : \mu = 8 \quad H_1 : \mu > 8$$

The variance is 4 and we get $\overline{X} = 9.5$ with sample size of 5. What can you conclude? (use 95% confidence interval)

References