IEOR165 Discussion
Week 1

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Outline

Sample Space and Events

Sample space Ω is a set of all possible outcomes and an event is a set of outcomes (Ross, 2007)

- Flipping a coin twice: $\Omega = \{HH, HT, TH, TT\}$. $A = \{HH, HT\}$ is the event that a head appears in the first time. Similarly, how do you describe $B = \{HH, HT, TH\}$?
- Rolling a dice: $\Omega = \{1, 2, 3, 4, 5, 6\}$. $A = \{2, 4, 6\}$ is the event that an even number appears on the roll.
- Lifetime of a car: $\Omega = [0, \infty)$. $A = (2, 6)$ is the event that the car lasts between two and six years.

Calculate Probabilities

(WAL 2.55) The probability that an American industry will locate in Shanghai is 0.7, the probability that it will locate in Beijing is 0.4, and the probability that it will locate in either Shanghai or Beijing or both is 0.8. What is the probability that the industry will locate
(a) in both cities? (b) in neither city?

Bayes' theorem

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

(IPM E1.14) A laboratory blood test is 95 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1 percent of the healthy persons tested. (That is, if a healthy person is tested, then, with probability 0.01, the test result will imply he has the disease.) If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that his test result is positive?

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Independent versus Mutually Exclusive

- Events are independent if the occurrence of one event does not influence (and is not influenced by) the occurrence of the other(s). E.g. when flipping coins, the result of the first flip does not influence the result of the last flip.
- Events are mutually exclusive if the occurrence of one event excludes the occurrence of the other(s). E.g. when flipping a coin, we can get a head or a tail but not both.

(PSES 3.47) Let A, B, C be events that $P(A) = 0.2, P(B) = 0.3, P(C) = 0.4$. Find the probability that at least one of the events A and B occurs if
(a) A and B are mutually exclusive;
(b) A and B are independent;
Find the probability that all of the events A, B, C occur if
(c) A, B, C are mutually exclusive;
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Solution: (a) 0.5; (b) 0.44; (c) 0; (d) 0.024.

Method of Moments

Given the sample: $X_1, \ldots, X_n$, we want to find estimators of parameters that charaterize the sample. The method of moment is, perhaps, the oldest method of finding point estimators, dating back to Karl Pearson in the late 1800s.

$$\mu_1 = E(X) = g_1(\theta_1, \ldots, \theta_p) \leftrightarrow \hat{\mu}_1 = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\vdots$$

$$\mu_p = E(X^p) = g_p(\theta_1, \ldots, \theta_p) \leftrightarrow \hat{\mu}_p = \frac{1}{n}\sum_{i=1}^{n} X_i^p$$

$$\Downarrow$$

$$\hat{\theta}_1 = h_1(\hat{\mu}_1, \ldots, \hat{\mu}_p)$$

$$\vdots$$

$$\hat{\theta}_p = h_p(\hat{\mu}_1, \ldots, \hat{\mu}_p)$$

Method of Moments

(SI E7.2.1) Let $X_1, \ldots, X_n$ be iid with normal distribution $N(\theta_1, \theta_2)$, estimate θ_1, θ_2 using method of moments.

Method of Moments

(SI 7.9) Let $X_1, \ldots, X_n$ be iid with probability density function:

$$f(x|\theta) = \frac{1}{\theta}, \quad 0 \leq x \leq \theta, \quad \theta > 0$$

Estimate θ using method of moments.