IEOR 151 – Service Operations Design and Analysis

Lab 8: Midterm Revew

October 13, 2016

In the first half of the semester, we have covered 2 out of the 4 main areas of the course:

- Service Quality Management
- Resource Allocation and Game Theory

In this lab session, we will look back and go through a quick review of the main topics in the 2 areas:

1. Point Gaussian Example

   A minimax hypothesis test:

   $$\min_{\delta(u)} \max_{\mu \in \{\mu_0, \mu_1\}} \mathbb{E}(L(\mu, \delta(X)))$$

   We have learned that the following decision rule is a minimax procedure (meaning that it solves the minimax optimization problem above):

   $$\delta(X) = \begin{cases} d_0 & \text{if } \bar{X} \leq \gamma^* \\ d_1 & \text{if } \bar{X} > \gamma^* \end{cases},$$

   where γ^* satisfies

   $$a(1 - \Phi(\sqrt{n}(\gamma^* - \mu_0)) = b\Phi(\sqrt{n}(\gamma^* - \mu_1))$$

   γ^* can then be solved via **binary search**. The binary search is conducted and the accuracy is defined with respect to γ^*

2. Newsvendor Problems

   We have seen a couple of varieties but the same concept governs the decision rule developed to address the risk. Demand (X) is random and there are costs associated with both "being over" and "going under" X. Based on our understanding and assumptions we make with regards to the uncertainty associated with X, we have devised specific decision strategies $\delta^*(X)$:

Type	$\delta^*(X)$
Standard Newsvendor	$F^{-1}(\frac{p}{p+q})$
Nonparametric Newsvendor	$\hat{F}^{-1}(\frac{p}{p+q}) = X_{\lceil n(\frac{p}{p+q}) \rceil}$
Linear Parametric Newsvendor	$\hat{\beta}Z$
Linear Parametric Newsvendor with Regularization	$\hat{\beta}_r Z$

When we face new problems, we can either 1) Design New Algorithms or 2) Reformulate/Reduce the New Problem to a simpler problem that existing algorithms can solve. In this section, we also taught you how to reformulate certain variants of the Newsvendor problem as a linear program (LP). For example, consider the nonparametric newsvendor problem:

$$\min_{\delta} \frac{1}{n} \sum_{i=1}^{n} (c_f + c_v \delta - p(\delta - X_i)^- + q(\delta - X_i)^+)$$

A simple reformulation gives us the following LP:

$$\begin{aligned} \min \quad & \frac{1}{n} \sum_{i=1}^{n} (c_f + c_v \delta + p s_i + q t_i) \\ \text{s.t.} \quad & s_i \geq -(\delta - X_i) \\ & t_i \geq (\delta - X_i) \\ & s_i, t_i \geq 0 \end{aligned}$$

3. Kidney Exchange

   We looked at both the edge formulation solution (the solution is a maximizer to an integer linear program (ILP)) and the cycle formulation solution (also an ILP). Solving either formulation of a large scale instance is computationally intensive (NP-Hard), but we have shown that the we can manually solve a small scale problem by noting that the clearing problem is solved by finding the *maximum weight union of disjoint cycles under the constraint that all cycles can have length less than or equal to a small constant L.*

4. Residency Matching

   In class, we have considered using a special case of the *Deferred Acceptance Algorithm/Gale-Shapley Algorithm.*

5. When we study matching markets, remember that assumptions may not hold in reality and a minor adjustment in assumptions can lead to drastically different results.

   Consider the following example: Match the applicants to the residency programs, and show intermediate steps of the algorithm.

   For this problem, suppose the applicant's preferences are given by:

1	**2**
B	A
A	B

   Suppose that each residency program has only 1 open position, and that the program's preferences are given by:

A	**B**
1	2
2	1

First attempt to run the Deferred Acceptance Algorithm. Then, reverse the role of hospitals and the residents.

How does the result change?

6. Adverse Selection Problems: Principal Agent Problems

    - Principal-agent models are situations in which there is a principal who wishes to delegate a task to an agent. The key settings are as follows: The agent has *private information* that is unknown to the principal. The challenge is to design contracts that will perform "properly" even when the agent has more information (i.e. in adverse selection problems). The main tradeoff concerns the amount of *information rent* the principal should pay in order to lead the agent to reveal its private knowledge.
    - The first best contracts represent the optimal contract to be offered in the ideal scenario where the Principal knows exactly which type the agent is
    - In reality, the principal agent does not know the agent's type and must optimize under the uncertainty and the corresponding optimization problem is as follows:

$$\begin{aligned} \max_{(t^I,q^I),(t^E,q^E)} \quad & \nu(S(q^E)-t^E)+(1-\nu)(S(q^I)-t^I) \\ \text{s.t.} \quad & t^I-\theta^I q^I-F\geq 0 \\ & t^E-\theta^E q^E-F\geq 0 \\ & t^I-\theta^I q^I-F\geq t^E-\theta^I q^E-F \\ & t^E-\theta^E q^E-F\geq t^I-\theta^E q^I-F \end{aligned}$$

    The first 2 constraints are the participation constraints and the last 2 constraints are the incentive compatibility constraints.

    - The optimality conditions (KKT conditions) give us the second-best transfers and productions levels respectively:

        (a) $q_2^I : S'(q_2^I)=\theta^I+\frac{\nu}{1-\nu}(\theta^I-\theta^E)$

        (b) $t_2^I : \theta^I q_2^I+F$ (Note that the inefficient agent makes no information rent)

        (c) $q_2^E=q_1^E$

        (d) $t_2^E=\theta^E q_2^E+(\theta^I-\theta^E)q_2^I+F=\theta^E q_1^E+(\theta^I-\theta^E)q_2^I+F$