

# IEOR 151 – Service Operations Design and Analysis

## Review Homework

12/03/2016

### 1. Service Staffing Problem

Formulate the following service staffing problem as a linear integer program. Do not solve!

You are hired by a bank to plan the workforce for their service operations. The planning horizon is 6 months from today. They currently have 12 tellers. The workforce hours required for the next 6 months are 1500, 1800, 1600, 2000, 1800, and 2200. You have the option of hiring trainees at the beginning of each month. However, each trainees should be hired a month before they can start working since they need one month training period. The training requires 80 hours of workforce hour from a regular teller for each trainee. At the end of each month, 10% of the workforce and trainees would quit. The direct labor cost is \$600 per month per teller, and \$300 per month per trainee. (A trainee would become a regular teller after the one month training period).

Formulate the ILP which can be solved to generate the optimal workforce plan (i.e. the number of trainees you hire each month and the number of available of tellers in each month)

*Variables:*

- $T_t$  - the number of trainees hired at the beginning of month  $t$  for  $t = 1, \dots, 6$
- $A_t$  - the number of tellers available at the beginning of month  $t$  for  $t = 1, \dots, 6$

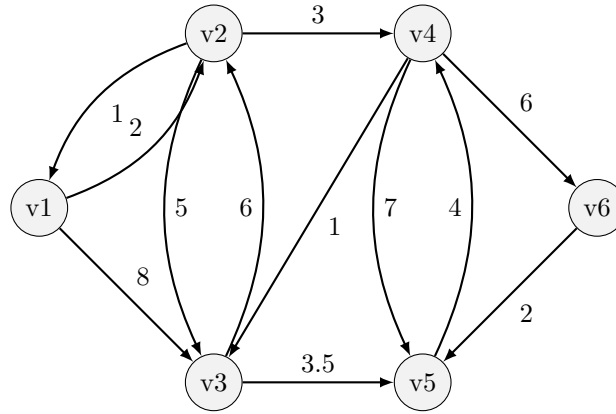
Assume the trainees cannot quit like regular tellers.

- 2. Service Staffing Problem II** Consider the previous problem again. However, consider the case where the bank can also receive temporary help from branches by offering temporary teller jobs to tellers under utilized in nearby branches. Due to the additional transportation cost needed, the temporary tellers are paid at \$900 per month. The number of available temporary tellers ( $B_t$ ) at time  $t$  are 10, 7, 11, 3, 2, 6. Reformulate the problem as an ILP.
- 3. Savings Algorithm** Consider the nodes described below, and note that the depot is located at node 0. Suppose we would like to solve this vehicle routing problem (VRP) using the savings algorithm:

Distance	Node 0	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Node 8	Demand
Node 0		26	15	20	7	25	16	24	29	0
Node 1			15	23	26	33	40	38	54	18
Node 2				24	13	20	27	35	43	26
Node 3					26	42	34	15	39	11
Node 4						18	14	31	32	30
Node 5							25	49	45	21
Node 6								32	20	16
Node 7									30	29
Node 8										37

- Solve for the constraint that each vehicle has a capacity of 200
- Solve for the constraint that each vehicle has a capacity of 100
- Which solution yielded a better solution?

4. **Deferred Acceptance Algorithm** Consider the following graph representation of a kidney exchange. Find the social welfare maximizing exchange under the constraint that all cycles can have length less than or equal to  $L = 3$ . (5 points)



5. Consider Hummingbird Cafe on Euclid in which customers arrive at a rate of 3. The service times are exponentially distributed with mean of 0.08 hours, and there is currently only 1 checkout line. Hint: The probability of a customer waiting in this  $M/M/1$  queue is given by  $\rho^2$  where  $\rho = \frac{\lambda}{\mu}$ ,  $\lambda$  is the arrival rate, and  $\mu$  is the service rate.
- What is the average number of customers in the queue?
  - What is the average time spent in the system? What is the average time spent waiting in line (before service)?
  - What is the probability of a customer having to wait?
  - Suppose the boss would like the probability of a customer having to wait be below 15%. Use the square root law to determine the number of servers he should hire for an  $M/M/s$  queue, in order to meet his objective?
6. (Bonus) A quick proof of the hint in the previous question:  
 Let  $p_k(t)$  be the probability that there are  $k$  customers in line at time  $t$ . In class, we showed that  $p_k$  is a geometric distribution (i.e.  $p_k = (1 - \rho)\rho^k$ ) and we will derive the result of the hint now. The probability that a customer is waiting =  $\sum_{k=2,3,\dots} p_k = 1 - p_0 - p_1 = 1 - (1 - \rho) - \rho(1 - \rho) = 1 - 1 + \rho^2 = \rho^2$ . Repeat the exercise to get a better understanding.