IEOR 151 – Midterm
October 19, 2016

Name:	
Overall:	/50

Instructions:

1. Show all your intermediate steps.
2. You are allowed a single 8.5x11 inch note sheet.
3. Calculators are allowed.
4. "Normal probability table" is given on last page.

1	/10
2	/10
3	/10
4	/20

1. Suppose you are the owner of a bread shop, and you would like to determine if your shop should expand its production capacity by investing in a new mixer with a large capacity. Investing in the new mixer will cost \$15,000, and will allow your shop to increase production capacity. If the current average demand is 220 per day, then investing in the new mixer will yield \$11,250 in additional profits. However, if the current average demand is 200 per day, the investment would be wasted. You have decided to use a minimax hypothesis testing approach to answer this question. As a first step, you record demand for goods over 10 days as follows:
   216, 210, 233, 233, 185, 219, 213, 245, 264, 264.

   (a) Assume that the demand for bread loaves per day is approximated by a Gaussian random variable with variance $\sigma^2 = 1600$. Using a binary search and z-table, compute the threshold for this hypothesis test γ^* to within an accuracy of ± 0.1.
   Hint: Use the following values for the minimax hypothesis test: $n = 10$, $\mu_0 = 200$, $\mu_1 = 220$, $\sigma^2 = 1600$, $L(\mu_0, d_0) = 0$, $L(\mu_0, d_1) = a = 15000$, $L(\mu_1, d_0) = b = 11250$, $L(\mu_1, d_1) = 0$. (8 points)
   Solution: Since $a > b$, $\gamma^* \in [210, 220]$

Step	γ	**LHS**	**RHS**
1	215	1755	3876
2	212.5	2416	3123
3	211.25	2800	2757
4	211.875	2604	2937

   $\gamma^* \in [211.25, 211.875]$

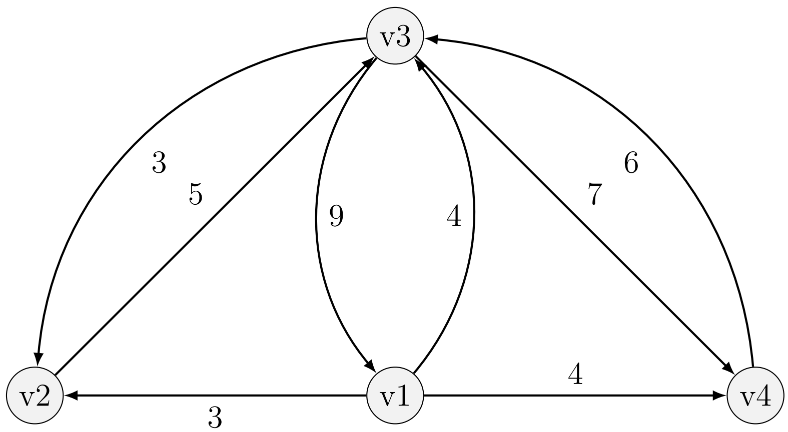
   (b) Should you invest in the new mixer? Explain your answer? (2 points)
   Solution
   Since $\bar{X} = 288.2$ and $\gamma^* < \bar{X}$, we should choose d_1 and invest in the new mixer.

2. Imagine that you are the owner of a bread shop and would like to determine the number of loaves of bread that should be made in the morning using a newsvendor model with production costs.

   (a) Suppose demand $X \sim \mathcal{N}(\mu = 220, \sigma^2 = 80)$, selling price is \$3.75 (r = 3.75), the per unit production cost is \$3.00 ($c_v = 3.00$), and the holding cost is \$1.00 ($q = 1.00$). What is the optimal inventory level? (5 points)
   Solution:
   $F(\delta^*) = \frac{r-c_v}{r+q} = \frac{.75}{4.75} = .1579$
   Hence, $\delta^* = 220 + (-1.0)\sqrt{80} = 211$

   (b) Now, suppose the manager measures the demand for bread for the past 20 days, and decides to use the nonparametric newsvendor model to solve the problem. The values of the demand, sorted into ascending order, are:
   245, 252, 190, 253, 231, 188, 202, 224, 257, 257, 193, 258, 257, 219, 244, 191, 214, 253, 243, 257.
   What is the optimal inventory level? (5 points)
   Solution:
   $\delta^* = X_{\lceil n(\frac{r-c_v}{r+q}) \rceil} = X_{\lceil 3.15 \rceil} = X_4 = 193$

3. Consider the following graph representation of a kidney exchange. Find the social welfare maximizing exchange under the constraint that all cycles can have length less than or equal to $L = 3$. (10 points)



Solution:

Cycle Label	**Cycles of** $L \leq 3$	**Cycle Weight**	**Disjoint Cycles**	**Weight**
A	$v1 \rightarrow v3 \rightarrow v1$	13	A	13
B	$v4 \rightarrow v3 \rightarrow v4$	13	B	13
C	$v2 \rightarrow v3 \rightarrow v2$	8	C	8
D	$v3 \rightarrow v1 \rightarrow v2 \rightarrow v3$	17	D	17
E	$v3 \rightarrow v1 \rightarrow v4 \rightarrow v3$	19	E	19

The Set of Disjoint Cycles {E } maximizes the social utility.

4. Imagine that you are the owner of a convenience store and would like to determine the number of candy bars to stock each day.

   (a) Suppose demand is modeled by an autoregressive (AR) model. In particular, given the demand on the t-th day which is X_t, the demand on the $(t+1)$-th day is $X_{t+1} = 0.8 \cdot X_t + W$, where $W \sim \mathcal{N}(\mu = 5, \sigma^2 = 10)$. Assume selling price is \$1.25 (r = 1.25), the per unit cost is \$1.00 ($c_v = 1.00$), and the holding cost is \$0.10 ($q = 0.10$). If the demand today ($t = 0$) is $X_0 = 7$, what is the optimal inventory level for tomorrow ($t = 1$)? (5 points)

   Solution:

   $X_1 = 0.8 * X_0 + W$.

   Essentially, the demand on day 1 is normally distributed with mean $\mu = .8 * 7 + 5 =$ and variance $\sigma = 10$.

   $F(\delta^*) = \frac{p - c_v}{p + q} = \frac{1.25 - 1}{1.25 + .1} = .1851$. Hence, $\delta^* = 10.6 + (-.09)\sqrt{10} = 7.75$

   (b) Again suppose demand is modeled by an autoregressive (AR) model $X_{t+1} = \alpha \cdot X_t + W$, where $W \sim \mathcal{N}(\mu, \sigma^2)$. However, now assume that α, μ, σ^2 are unknown. Instead, suppose we have measured demand over the days $X_0, \ldots, X_n$. Assume selling price is \$1.25 (r = 1.25), the per unit cost is \$1.00 ($c_v = 1.00$), and the holding cost is \$0.10 ($q = 0.10$). Formulate a linear programming (LP) model that can be used to compute an inventory level for X_{n+1}. (5 points)

   Solution:

   With unknown α, μ, σ^2 and n samples, we should consider using regression to fit a linear model under the AR(1) formulation. Our goal is to obtain estimates of a, b, where $\hat{X}_t = aX_{t-1} + b$ is the best linear model such that $\delta_t^* = aX_{t-1} + b$, and we can reformulate the problem as a linear program as follows:

$$
\begin{aligned}
\min_{a,b} \quad & \frac{1}{n} \sum_{t=1}^{n} (c_f + c_v \delta_t + p s_t + q t_t) \\
\text{s.t.} \quad & s_t \geq -(\delta_t - X_t) \\
& \delta_t = aX_{t-1} + b \\
& t_t \geq (\delta_t - X_t) \\
& s_t, t_t \geq 0 \\
& x_0 \ \text{ given}
\end{aligned}
$$

Normal probability table

	Second decimal place of Z									
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

*For $Z \geq 3.50$, the probability is greater than or equal to 0.9998.