IEOR 151 – Service Operations Design and Analysis

Homework 3 (Due 11/11/2016)

10/28/2016

1. In class, we looked at both the P-median and P-center model. The P-center model minimizes the maximum over all nodes distance between a demand node and the facility to which it is assigned, while the P-median model simply minimizes the demand weighted average distance. The constraints are the same for both problems. Therefore, we can use a **combined model** to find the tradeoff between the 2 objectives (minimizing the average and the maximum distances). Suppose we have two solutions as shown in the table below.

Solution	Avg. Distance	Max. Distance
1	100	600
2	300	325

   - What should be the value of W if the weighted objective is - W(Avg. Dist) + (1-W)(Max. Dist)?
     Answer:

     $$
     \begin{aligned}
     100W + 600(1 - W) &= 300W + 325(1 - W) \\
     275(1 - W) &= 200W \\
     W &= \frac{75}{125} = 0.58
     \end{aligned}
     $$

   - Use the weighting method to find a new solution on the tradeoff curve for the table below:

Solution	Avg. Distance	Max. Distance
1	100	600
2	300	325
3	250	450

   Answer: Use a quadratic function to fit through the 3 points - (100, 600), (300, 325) & (250, 450) . A new solution can be selected on the tradeoff curve/pareto frontier.

2. Practitioners are often interested in achieving multiple objectives with varying priorities (different from question 1). Consider the following network:
   Numbers shown in square beside the nodes are the demand per week at the node.
   Consider the hierarchical objective problem of **first** finding the minimum number of facilities needed to cover all nodes within a coverage distance of 23, and then, from among all of the alternate optima for this problem, **second** finding the solution that maximizes the number of demands that are covered two or more times.
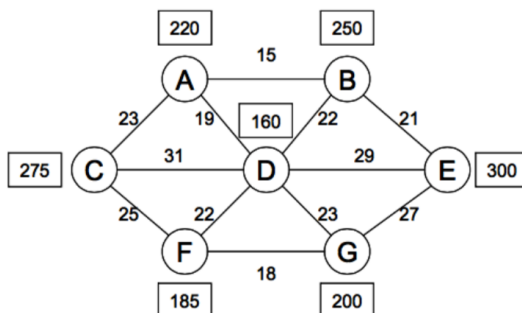
Define the following notation:

Inputs:

(a) J: Set of demand nodes

(b) K: Set of candidate nodes (same as the set of demand nodes in this case)

(c) a_{jk}: 1 if demand node j is covered by a facility located at node k; 0 otherwise

(d) h_j: demand at node j

(e) W: weight on the primary objective of minimizing the number of facilities

Decision Variables:

(a) X_k: 1 if facility is located at candidate site k; 0 if not

(b) Z_j: 1 if demand node j is covered 2 or more times; 0 if not

- Formulate the hierarchical objective of FIRST minimizing the number of facilities needed and SECOND maximizing the number of demands that are covered 2 or more times. Note that this should be a single function. This should be formulated in terms of notation.
  Answer: $\min W \sum_{k \in K} X_k - \sum_{j \in J} h_j Z_j$

- Formulate the constraints for this model including any integrality constraints. Explain the constraints using notations as well as words.
  Answer:

$$\text{Subject to:} \quad \sum_{k \in K} a_{jk} X_k - Z_j \geq 1 \quad \forall\, j \in J$$
$$X_k \in 0, 1 \qquad\qquad \forall k \in K$$
$$Z_j \in 0, 1 \qquad\qquad \forall j \in J$$

- Do the Z_j variables have to be constrained to be binary, or can you simply use a constraint of the following form: $0 \leq Z_j \leq 1 \forall j$
  Answer: No, you do not. Once you require the location variables to be integer, the quantity $\sum_{k \in K} a_{jk} X_k$ will be integer and then the value of Z_j will be 0 if $\sum_{k \in K} a_{jk} X_k = 1$ and 1 if $\sum_{k \in K} a_{jk} X_k > 1$ since all demands are positive.

- How large should W be to ensure that the combined objective function of part (a) first minimizes the number of facilities and then selects the solution that maximizes the number of multiple covered demands from among the alternate optima? Briefly justify your answer. What you want is the smallest possible value of W.
  Answer: W should be greater than the sum of the demands so that adding an extra facility will hurt the first term of the objective function more than it can possibly benefit the second term

3. In class, we considered 3 facility location models which did not take into account the fixed charged associated with facility location models. The following question considers both transportation and location cost. Kaminsky-Shen company is in the process of planning for new production facilities and developing a more efficient distribution system design. At present, they have one plant at St. Louis with a capacity of 30,000 units. But because of increased demand, management is considering four potential new plant sites: Detroit, Denver, Toledo, and Kansas City. The transportation tableau below summarizes the projected plant capacities, the cost per unit of shipping from each plant to each destination , and the demand forecasts over a one-year planning horizon:

	Destination			
	Boston	Atlanta	Houston	**Capacities**
Detroit	5	2	3	10,000
Toledo	4	3	4	20,000
Denver	9	7	5	30,000
Kansas City	10	4	2	40,000
St. Louis	8	4	3	30,000
Demand	30,000	20,000	20,000	

Suppose that the fixed costs associated with the new plants are $175,000, $300,000, $375,000, and $500,000 for Detroit, Toledo, Denver, and Kansas City respectively.
The company would like to minimize the total cost of plant construction and distribution of goods.

- Develop a 0-1 mixed integer linear programming model of this model. (Do not attempt to solve)
  answer:
  Decision Variables: x_i: binary variable - 1 if plant i is constructed
  Assignment Variables: Y_{ij}: amount of goods shipped from plant i to demand j
  Parameters:

  (a) f_i: fixed cost of building plant i

  (b) c_{ij}: per unit cost of shipping from plant i to demand j

  (c) a_i: capacity at node i

  (d) b_j: demand at node j

$$\min \sum_{j=1}^{4} f_i x_i + \sum_{i=1}^{3} c_{ij} Y_{ij}$$

$$\sum_{j=1}^{3} Y_{ij} \le a_i x_i \ \forall i$$

$$\sum_{i=1}^{5} Y_{ij} \ge b_j \ \forall j$$

$$Y_{ij} \ge 0 \ \forall i \ \forall j$$

$$X_i \in \{0, 1\} \ \forall i$$

- Modify your formulation in part a to account for the policy restriction that one plant but not two must be located in Kansas City or in Toledo. (Do not attempt to solve)
  answer:
  Add the following constraint: $x_2 + x_4 \le 1$

- Modify your formulation in part a to account for the policy restriction that at most three plants can be located in Toledo, Denver, Detroit, and St. Louis. (Do not attempt to solve)
  answer:
  Add the following constraint: $x_1 + x_2 + x_3 + x_5 \le 3$

- Suppose that there are two possible sizes for the Denver plant, the one mentioned earlier with a capacity of 30,000 and a cost of $375,000, and another with a capacity of 60,000 and a cost of $550,000. Modify your formulation in part a to account for this consideration.
  answer:
  Augment the set of plant locations with 6 to denote the new Denver plant and add the following constraint: $x_3 + x_6 \leq 1$

4. Consider the nodes described below, and note that the depot is located at node 0. Solve this vehicle routing problem (VRP) using the savings algorithm, for the constraint that each vehicle has a capacity of 25 units (meaning it can carry less than or equal to 25 units). Please list the nodes visited on each route, the demand associated with that route, and the route length. (10 points)

Distance	Node 0	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Demand
Node 0								0
Node 1	6.23							10
Node 2	4.48	5.66						12
Node 3	2.83	6.16	2.83					6
Node 4	4	8	5.66	2.83				13
Node 5	5	8.24	8.06	5.39	3			14
Node 6	2	7.47	6	7.47	7.47	7.47		7
Node 7	5.24	9.16	3.62	7.07	2.62	3.5	2.16	10

answer:
First, calculate the savings:

$$
\begin{aligned}
s_{12} &= 5.03 \\
s_{13} &= 2.9 \\
s_{14} &= 2.23 \\
s_{15} &= 2.99 \\
s_{16} &= 0.76 \\
s_{17} &= 2.31 \\
s_{23} &= 4.48 \\
s_{24} &= 2.82 \\
s_{25} &= 1.42 \\
s_{26} &= 0.48 \\
s_{27} &= 6.1 \\
&\cdots
\end{aligned}
$$

Then sort the savings from large to small and we get:

$$s_{57}$$
$$s_{47}$$
$$s_{27}$$
$$s_{45}$$
$$s_{67}$$
$$s_{12}$$
$$s_{23}$$
$$s_{34}$$
$$s_{15}$$
$$s_{13}$$
$$s_{24}$$
$$s_{35}$$
$$s_{17}$$
$$s_{14}$$
$$s_{25}$$
$$s_{37}$$
$$s_{16}$$
$$s_{26}$$
$$s_{36}$$
$$s_{46}$$
$$s_{36}$$

With a capacity ≤ 25. We get the following routes

Route	Demand	Length
0 - 5- 7 - 0	$14 + 10 = 24$	13.75
0 - 1 - 2 - 0	$10 + 12 = 22$	16.37
0 - 3 - 4 - 0	$6 + 13$	$2.83 + 2.83 + 4 = 9.66$
0 - 6 - 0	7	4

5. Solve a P-center problem using the greedy heuristic algorithm: allocate 2 facilities among 7 demand nodes (demand nodes set and candidate sites set are the same). The demand and distance information is given in the figure below.
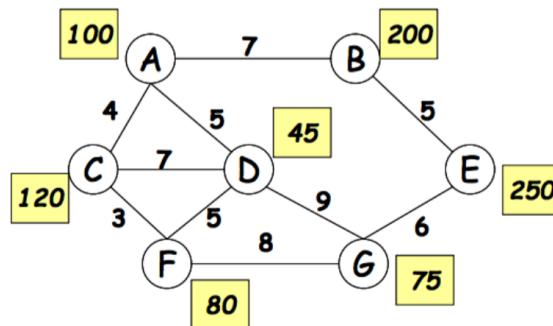
answer:
There are many solutions as you are randomly selecting the first facility to locate. Here is one solution:

1.

2.