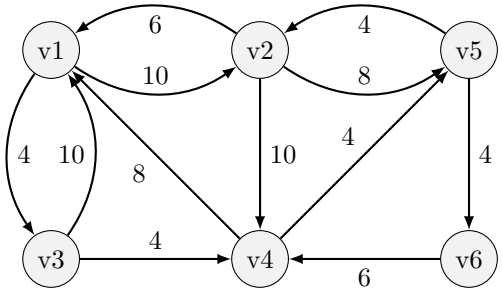
IEOR 151 – Service Operations Design and Analysis

Homework 2 (Due 10/16/2015)

October 13, 2016

1. Consider the following graph representation of a kidney exchange. Find the social welfare maximizaing exchange under the constraint that all cycles can have length less than or equal to L = 3. (5 points)



Solution:

Cycle Label	**Cycles of $L \leq 3$**	**Cycle Weight**	**Disjoint Cycles**	**Weight**
A	$v1 \to v2 \to v1$	16	A, G	30
B	$v1 \to v3 \to v1 \to v1$	14	B, E	26
C	$v1 \to v2 \to v4 \to v1$	28	B, F	32
D	$v1 \to v3 \to v4 \to v1$	16	B, G	28
E	$v2 \to v5 \to v2$	12	C	28
F	$v2 \to v4 \to v5 \to v2$	18	D, E	28
G	$v5 \to v6 \to v4 \to v5$	14		

The Set of Disjoint Cycles {B, F} maximizes the social utility.

2. Consider the case there are 4 coaches {1, 2, 3, 4 } and 4 athletes {A, B, C, D } on a track and field team. Match the coaches to the athletes based on the listed preferences. Show intermediate steps of the algorithm. The athletes' preferences are given by:

A	**B**	**C**	**D**
1	4	2	3
3	3	3	4
2	2	1	2
4	1	4	1

Suppose that each coach can only mentor 1 athlete, and the coaches's preferences are:

1	**2**	**3**	**4**
A	A	A	A
B	D	C	B
C	C	B	C
D	B	D	D

First, consider the case where the coaches can choose first. Then, consider the case where the athletes can choose first. Does the matching change? Solution: When the coaches choose first:

1	**2**	**3**	**4**
A	D	C	B

No one is unmatched. When the athletes choose first:

A	**B**	**C**	**D**
1	4	2	3

No one is unmatched again.
We note that the matching changes as the order of selection changes.

3. Match the applicants to the residency programs, and show intermediate steps of the algorithm. (5 points)
For this problem, suppose the applicant's preferences are given by:

Anil	**Alper**	**Candi**	**Rhonda**
Hopkins	General	General	Temple
General	Hopkins	Temple	Hopkins
Temple	Temple	Hopkins	General

Suppose that each residency program has only 1 open position, and that the program's preferences are given by:

General	**Hopkins**	**Temple**
Candi	Rhonda	Rhonda
Rhonda	Alper	Candi
Anil	Candi	Anil
Alper	Anil	Alper

Solution:

General	**Hopkins**	**Temple**
~~Alper~~	~~Anil~~	Rhonda
Candi	Alper	

Anil is unmatched.

4. Suppose Apple, Inc would like to purchase processors from Samsung Electronics Co. Apple's utility for the processors is given by $S(q) = 550\sqrt{(1+q)}$. The fixed costs for Samsung Electronics Co. are \$5,500, and if Samsung is inefficeint (efficient) then its marginal costs are 2.75 (2.00). Assume that Apple, Inc believes that there is a 25% chance that Samsung Electronics Co. is efficient.

   (a) What are the first-best production levels? (2 points)
   $S'(q_1^I) = \theta^I \Rightarrow q_1^I : \frac{550}{2\sqrt{1+q_1^I}} = \theta^I \Rightarrow q_1^I = 9,999$
   Similarly, equating marginal utility to marginal costs for the efficient distributor gives:
   $S'(q_1^E) = \theta^E \Rightarrow q_1^E : \frac{550}{2\sqrt{1+q_1^E}} = \theta^E \Rightarrow q_1^E = 18905.25$

   (b) What are the contracts to implement the first-best production levels? (2 points)
   Solution:
   These contracts allow for zero information rent, meaning that the inefficient distributor should be offered the contract:

$(q_1^I = 9,999, t_1^I = \theta_1^I q_1^I + F = 2.75 * 9999 + 5500 = 32,997.25)$
and the efficient distributor should be offered the contract:
$(q_1^E = 18905.25, t_1^E = \theta_1^E q_1^E + F = 2 * 2749 + 5500 = 43,310.5)$

(c) How much profit would Samsung Electronics Co. make if Apple, Inc offers a menu of contracts $\{(q_1^I, t_1^I), (q_1^E, t_1^E)\}$ (1 point)
If Samsung is inefficient, Profit = 0.
If Samsung is efficient, Profit $= t_1^I - \theta^E q_1^I - F = 32,997.25 - (2)(9999) - 5500 = 7499.25$

(d) What are the second-best production levels? (2 points)
Solution:
The production level for the efficient agent remains unchanged $q_2^E = q_1^E = 18905.25$.
The production level for the inefficient agent decreases to:

$$q_2^I : S'(q_2^I) = \theta^I + \frac{\nu}{1-\nu}(\theta^I - \theta^E) \Rightarrow q_2^I : \frac{550}{2\sqrt{1+q_2^I}} = 2.75 + \frac{1}{3}(.75) \Rightarrow q_2^I = 8401$$

(e) What is the menu of contracts for the second-best production levels? (2 points)
Solution:
The transfer for the efficient agent is:
$t_2^E = \theta^E q_2^E + (\theta^I - \theta^E) q_2^I + F = (2)(218905.25) + (0.75)(8401) + 5500 = 49611.8$
and the transfer for the inefficient agent is:
$t_2^I = \theta^I q_2^I + F = (2.75)(8401) + 5500 = 28605$
Hence, the menu of contracts are $\{(q_2^E = 18905.25, t_2^E = 49611.8)(q_2^I = 8401, t_2^I = 28605)\}$

(f) What is the information rent of an efficient Samsung Electronics Co. for the menu of contracts for the second-best production levels? Is this higher or lower than the profit gained for the menu of contracts for the first-best production levels? (2 points)
Solution:
Information rent $= (\theta^I - \theta^E) q_2^1 = 0.75(8401) = 6301 < 7499.25$. Hence, Samsung gathers less profit/Information Rent with the second best contract.