IEOR 151 – Lecture 11
Adverse Selection

1 Sandwich Example

Consider the following hypothetical situation: A company is holding a picnic and would like to purchase grilled cheese sandwiches with tomatoes and swiss cheese on sourdough bread with a toasted parmesan crust. The company can use every grilled cheese sandwich that is made; however, there are diminishing returns to the value of each additional sandwich that gets made. Furthermore, the restaurant making the sandwiches is either efficient or inefficient at making the grilled cheese sandwiches; however, the company does not know the *type* of the restaurant. Here, efficiency is measured with respect to the cost of making each sandwich in terms of raw ingredients, energy inputs, and equipment purchases. Given this situation, it is interesting to understand how the company should structure the contract used to hire the restaurant.

This example is part of a larger set of situations. Principal-agent models are situations in which there is a principal (e.g., manager) that wishes to delegate a task to an agent (e.g., employee). Our example of a company (principal) hiring a restaurant (agent) follows this pattern. In this example, there is also an element of information asymmetry: The agent has *private knowledge* that is unknown to the principal. Situations in which the agent has more information is known as *adverse selection*, and it makes the problem of designing a contract more difficult. This complicates the design problem because an efficient contract is one that leads to the agent revealing its private knowledge, but this will require giving up some *information rent* to the agent.

2 Principal-Agent Model with Adverse Selection

Suppose the principal contracts the agent to product q units of a good. The value to the principal of the q units of the good is given by the function $S(q)$ where $S(0) = 0$, $S' > 0$, and $S'' < 0$; this means that the marginal value of the good is positive (more good produced is better) but strictly decreasing (diminishing returns).

In this model, the production cost of the agent is unobservable to the principal. However, a number of facts are known to both. Both know the value of fixed costs F, the cost function of the agent $C(q, \theta) = \theta q + F$ where θ is the marginal cost, and that the agent is either inefficient ($\theta = \theta^I$) or efficient ($\theta = \theta^E$) with $\theta^I > \theta^E$ (higher marginal cost for inefficient type). The agent knows its own type, but the principal does not. Instead, the principal and agent know the probability an agent is efficient $\mathbb{P}(\theta = \theta^E) = \nu$ or inefficient $\mathbb{P}(\theta = \theta^I) = 1-\nu$. The last feature of this model is which decisions are made. The principal gets to design a menu contract by specifying multiple production levels and associated payments (or transfers) t for that given production level. We will use the tuple (q, t) to represent one possible contract in a menu, and the menu will consist of multiple tuples.

2.1 Sandwich Example

We will assume that the princpal's value is given by $S(q) = 10\sqrt{q}$, fixed costs for the agent are $F = 20$ dollars, the marginal costs for an inefficient agent are $\theta^I = 0.30$, and the marginal costs for an efficient agent are $\theta^H = 0.25$. Furthermore, we assume that the agent is efficient with probability $\nu = 0.20$ and inefficient with probability $1 - \nu = 0.80$.

3 First-Best Production Levels

Pretend for a moment that the principal does know the type of the agent. Then, the principal should design the contract so that the principal's marginal utility of a good equals the agent's marginal cost. This means that we should pick $q_1^I : S'(q_1^I) = \theta^I$ and $q_1^E : S'(q_1^E) = \theta^E$. However, the agents will not participate in a contract if offered a transfer t such that they would lose money. Hence, we have the following *participant constraints* that the transfers in the contract must satisfy

$$
\begin{aligned}
t^I - \theta^I q^I - F &\geq 0 \\
t^E - \theta^E q^E - F &\geq 0
\end{aligned}
$$

Combining everything, our menu of contracts is the following: If the agent is $\theta = \theta^I$ then offer the contract $(q_1^I, \theta^I q_1^I + F)$, and if the agent is $\theta = \theta^E$ then offer the contract $(q_1^E, \theta^E q_1^E + F)$. What is interesting about these contracts are that the agents make no profit (meaning their transfer is equal to their cost) regardless of their type.

3.1 Sandwich Example

For our example, the principal's marginal value of a sandwich is given by

$$S'(q) = \frac{10}{2\sqrt{q}}.$$

Equating marginal value to marginal costs for the inefficient restaurant gives

$$
\begin{aligned}
q_1^I : S'(q_1^I) = \theta^I \Rightarrow q_1^I &: \frac{10}{2\sqrt{q_1^I}} = 0.30 \\
&\Rightarrow q_1^I = 277.
\end{aligned}
$$

Similarly equating marginal value to marginal costs for the efficient restaurant gives

$$q_1^E : S'(q_1^E) = \theta^E \Rightarrow q_1^E : \frac{10}{2\sqrt{q_1^E}} = 0.25$$
$$\Rightarrow q_1^E = 400.$$

Given these production levels, we should offer the inefficient restaurant a contract of

$$(q_1^I = 277, t_1^I = \theta^I q_1^I + F = 0.30 \cdot 277 + 20 = 103.10),$$

and we should offer the efficient restaurant a contract of

$$(q_1^E = 400, t_1^E = \theta^E q_1^E + F = 0.25 \cdot 400 + 20 = 120.00).$$

4 Incentive Feasible Contracts

Now, we return to the situation where the principal does not know the type of the agent. The first idea is that the principal might offer a menu of contracts $\{(q_1^I, t_1^I), (q_1^E, t_1^E)\}$, and hope that the inefficient agent chooses the contract (q_1^I, t_1^I) and the efficient agent chooses the contract (q_1^E, t_1^E). However, this is not what happens. It is in the interests of the efficient agent to pretend to be inefficient and accept the contract (q_1^I, t_1^I), because this leads to a profit for the efficient agent. This occurs because for a fixed production level, say q_1^I, the efficient agent will have lower costs than the inefficient agent $\theta^E q_1^I + F < \theta^I q_1^I + F$. And since the contract is designed so that $t_1^I - \theta^I q_1^I - F = 0$, this implies that $t_1^I - \theta^E q_1^I - F > 0$ which is a profit for the efficient agent pretending to be inefficient.

In the sandwich example, the efficient agent accepting the (q_1^I, t_1^I) contract makes a profit of

$$t_1^I - \theta^E q_1^I - F = 103.10 - 0.25 \cdot 277 - 20 = 13.85,$$

which is greater than the zero profit made when the efficient agent accepts the contract (q_1^E, t_1^E).

Ideally, we would like to design our menu of contracts so that each agent picks the contract that is designed for their type. In other words, we would like the agents to be *truthful* about their type. This feature of being truthful is also known as a contract that is *strategy proof* or *incentive compatible*, and it is described by the following *incentive compatibility constraints*

$$t^E - \theta^E q^E - F \geq t^I - \theta^E q^I - F$$
$$t^I - \theta^I q^I - F \geq t^E - \theta^I q^E - F.$$

The intuition is that the profit that the efficient (inefficient) agent makes when selecting the contract designed for the efficient (inefficient) should be greater than or equal to the profit the efficient agent makes when selecting the contract designed for the inefficient (efficient) agent.

4.1 Information Rents

The amount of profit that an agent makes is known as information rent. For the efficient and inefficient agents, the information rent is defined by

$$U^E = t^E - \theta^E q^E - F$$
$$U^I = t^I - \theta^I q^I - F.$$

Even when a contract is designed taking the incentive compatibility constraints into account, some amount of information rent is generally given to the efficient agent. To see why, imagine that the efficient agent were to pretend to be inefficient. Then, its payment would be

$$U^E = t^E - \theta^E q^E - F \geq t^I - \theta^E q^I - F = t^I - \theta^I q^I - F + (\theta^I - \theta^E)q^I = U^I + (\theta^I - \theta^E)q^I.$$

And so if the contract is designed so that $U^I = 0$, there is still some nonzero information rent of $(\theta^I - \theta^E)q^I$. In the sandwich example, this value is $(\theta^I - \theta^E)q^I_1 = (0.30 - 0.25) \cdot 277 = 13.85$, which was the profit of the efficient agent; however, the rent can be decreased by decreasing the value of q_I. How to do so is an important problem.

4.2 Optimally Designing the Contract

Given that the efficient agent must receive some information rent, an interesting question to ask is how to design the contract so as the minimize the rent given and increase the overall efficiency for the principal. Given our model, we can pose this as an optimization problem

$$\begin{aligned} \max_{(t^I, q^I), (t^E, q^E)} \quad & \nu(S(q^E) - t^E) + (1 - \nu)(S(q^I) - t^I) \\ \text{s.t.} \quad & t^I - \theta^I q^I - F \geq 0 \\ & t^E - \theta^E q^E - F \geq 0 \\ & t^E - \theta^E q^E - F \geq t^I - \theta^E q^I - F \\ & t^I - \theta^I q^I - F \geq t^E - \theta^I q^E - F. \end{aligned}$$

Recall that the first two constraints are the participation constraints, and the last two constraints are the incentive compatibility constraints.

This optimization problem can be solved from its optimality conditions, and the corresponding solutions are known as the second-best production levels. We summarize the results here. The production levels for the efficient agent remains the same as the first-best production levels, that is $q^E_2 = q^E_1$. The production level for the inefficient agent is decreased to

$$q^I_2 : S'(q^I_2) = \theta^I + \frac{\nu}{1 - \nu}(\theta^I - \theta^E).$$

The corresponding second-best transfers are given by

$$t^E_2 = \theta^E q^E_2 + (\theta^I - \theta^E)q^I_2 + F = \theta^E q^E_1 + (\theta^I - \theta^E)q^I_2 + F,$$

and $t^I_2 = \theta^I q^I_2 + F$. Here, only the efficient agent gets a strictly positive information rent $U^E_2 = (\theta^I - \theta^E)q^I_2$.

4.3 Sandwich Example

Returning to the sandwich example, the second-best production levels for the inefficient agent are given by

$$q_2^I : S'(q_2^I) = \theta^I + \frac{\nu}{1-\nu}(\theta^I - \theta^E) \Rightarrow q_2^I : \frac{10}{2\sqrt{q_2^I}} = 0.30 + \frac{0.20}{1-0.20}(0.30 - 0.25)$$
$$\Rightarrow q_2^I = 256.$$

The transfer for the efficient agent's production level is

$$t_2^E = \theta^E q_2^E + (\theta^I - \theta^E) q_2^I + F = 0.25 \cdot 400 + (0.30 - 0.25) \cdot 256 + 20 = 132.80,$$

and the transfer for the inefficient agent's production level is

$$t_2^I = \theta^I q_2^I + F = 0.30 \cdot 256 + 20 = 96.80.$$

Summarizing, the menu of contracts for the second-best levels of production is

$$\{(q_2^I = 256, t_2^I = 96.80), (q_2^E = 400, t_2^E = 132.80)\},$$

and the information rent extracted by the efficient agent is $U_2^E = (\theta^I - \theta^E) q_2^I = (0.30 - 0.25) \cdot 256 = 12.80$, which is lower than the information rent $U_1^E = 13.85$ when using the menu of contracts for the first-best levels of production.

5 More Reading

The material in this lecture follows that of the textbook *Theory of Incentives* by Laffont and Martimont. More details about this model can be found there.