IEOR 151 – Service Operations Design and Analysis

Practice Questions 2015

October 19, 2015

1. A newsboy would like to select the optimal level of inventory of newspapers using a newsvendor model without production costs. Here, demand $X \sim \mathcal{N}(120, 60)$ is in units of newspapers, holding costs are \$0.9 per newspaper, and the sale price is \$1.5 per newspaper. What is the optimal inventory level? (4 points)
   Solution:
   $F(\delta^*) = \frac{p}{p+q} = \frac{1.5}{1.5+0.9} = 0.625$
   Given that X is normally distributed, we solve δ^* as follows: $\frac{\delta^*-120}{\sqrt{60}} = 0.32$. Hence, $\delta^* = 122.479(123)$ units.

   - Now, suppose the newsboy measures the demand of newspapers for the past 10 days, and decides to use the nonparametric newsvendor model to solve the problem. The values of the demand, sorted into ascending order, are: 80, 84, 97, 101, 112, 119, 124, 131, 141, 162.
     Solution:
     Hint: Consider the sample distribution:

     $$\hat{F}(z) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}(z \leq X_i)$$

     Solution:
     Considering the empirical CDF, we adopt the following approach - $n\frac{p}{p+q} = 10(\frac{1.5}{1.5+0.9}) = 6.25$. Then the solution corresponds to the $X_{\lceil n(\frac{p}{p+q})\rceil} = X_7$. Hence, the newsboy should purchase $X_7 = 124$ newspapers.

2. Suppose you are the manager of the bakery and would like to determine the number of muffins that should be made in the morning using the nonparametric newsvendor model. Assume the selling price is \$2.5 ($r = 2.5$), the per unit production cost is \$1.25 ($c_v = 1.25$) and the holding cost is \$0.65 ($q = 0.65$). Now, suppose the manager measures the demand of muffins for the past 20 days, and the values of the demand, sorted into ascending order, are: 16, 22, 23, 31, 33, 37, 42, 51, 53, 62, 64, 67, 69, 70, 73, 73, 75, 82, 83, 91.
   Solution:
   Hint: Consider the sample distribution:

   $$\hat{F}(z) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}(z \leq X_i)$$

   Solution:
   Considering the empirical CDF, we adopt the following approach - $n\frac{r-c_v}{r+q} = 20\frac{2.5-1.25}{2.5+0.65} = 7.937$. Then the solution corresponds to the $X_{\lceil n(\frac{r-c_v}{r+q})\rceil} = 8$. Hence, the manager should produce $X_8 = 51$ muffins.

3. Suppose you are the product manager of the Expedia, and you would like to determine if the company should purchase a new electronic system to manage online hotel bookings. Purchasing the system will cost \$490, and similar website around the world have found that using the electronic system leads to an average of 28 mistakes (e.g. lost sales opporunities) per day. If the current error rate is 36 mistakes

per day, then purchasing the new system will have a net savings of $360 over the span of one year. You have decided to use a minimax hypothesis testing approach to answer this questions. As a first step, you record the number of mistakes made over 10 days: 16, 49, 24, 24, 28, 30, 31, 38, 43, 23.

- Assume that the number of mistakes per day is approximated by a Gaussian random variable with variance $\sigma^2 = 36$. Using a binary search and z-table, compute the threshold for this hypothesis test γ^* to within an accuracy of ± 0.1 (4 points)
  Hint: Use the following values for the minimax hypothesis test: $n = 10$, $\mu_0 = 28$, $\mu_1 = 36$, $\sigma^2 = 36$, $L(\mu_0, d_0) = 0$, $L(\mu_0, d_1) = a = 490$, $L(\mu_1, d_0) = b = 360$, $L(\mu_1, d_1) = 0$
  Solution:
  Consider the following comparison and recall that the goal is the select $\gamma*$ such that

$$a(1 - \Phi(\frac{\sqrt{n}(\gamma^* - \mu_0)}{\sigma}) = b\Phi(\sqrt{n}\frac{(\gamma^* - \mu_1)}{\sigma})$$

$$490(1 - \Phi(\frac{\sqrt{10}(\gamma^* - 28)}{\sqrt{36}}) = 360\Phi(\sqrt{10}\frac{(\gamma^* - 36)}{\sqrt{36}})$$

  Since $a > b$, binary search should be conducted on $[32, 36]$ and the best first guess is 34
  Note, the required accuracy concerns gamma rather than the difference between LHS and RHS.
  Using binary search, we obtain $\gamma^* \in (32.0625, 32.125)$

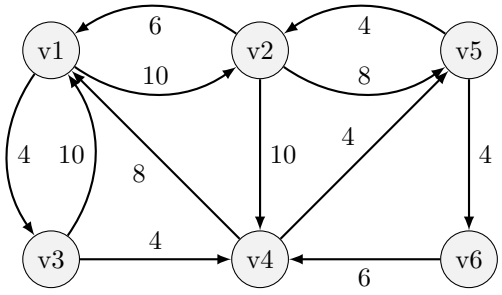
Step	γ	**LHS**	**RHS**
1	34	0.392	52.884
2	33	2.0	20.556
3	32.5	4.361	11.844
4	32.25	6.125	8.604
5	32.125	7.35	7.452
6	32.0625	7.938	6.912

- Should the manager purchase the new electronic system? Explain your answer (2 points)
  Solution:
  $\bar{X}$ (Sample Mean) $= 30.6 < \gamma^*$.
  Given the decision rule

$$\delta(X) = \begin{cases} d_0 & \text{if } \bar{X} \leq \gamma^* \\ d_1 & \text{if } \bar{X} > \gamma^* \end{cases}$$

  The manager chooses d_0 and does not purchase the new system.

4. Consider the following graph representation of a kidney exchange. Find the social welfare maximizaing exchange under the constraint that all cycles can have length less than or equal to L = 3. (5 points)



  Solution:
  First, we list all cycles of length $L \leq 3$ and compute the weight of these cycles. Next, we determine all sets of disjoint cycles and compute their weight. Lastly, the solution is the set of disjoint cycles with maximal weight. The steps are shown below, and the social welfare maximizing exchange is the set of disjoint cycles C, F

Cycle Label	Cycles of $L \leq 3$	Cycle Weight	Disjoint Cycles	Weight
A	$v1 \to v2 \to v1$	16	A, G	30
B	$v1 \to v2 \to v4 \to v1$	28	B	28
C	$v1 \to v3 \to v1$	14	C, E	26
D	$v1 \to v3 \to v4 \to v1$	16	C, F	32
E	$v2 \to v5 \to v2$	12	C, G	28
F	$v2 \to v4 \to v5 \to v2$	18	D, E	28
G	$v4 \to v5 \to v6 \to v4$	14	E	12
			F	18
			G	14

5. Match the applicants to the residency programs, and show intermediate steps of the algorithm. (5 points)
For this problem, suppose the applicant's preferences are given by:

Anil	Alper	Candi	Rhonda
Hopkins	General	General	Temple
General	Hopkins	Temple	Hopkins
Temple	Temple	Hopkins	General

Suppose that each residency program has only 1 open position, and that the program's preferences are given by:

General	Hopkins	Temple
Candi	Rhonda	Rhonda
Rhonda	Alper	Candi
Anil	Candi	Anil
Alper	Anil	Alper

Solution:

General	Hopkins	Temple
~~Alper~~	~~Anil~~	Rhonda
Candi	Alper	

Anil is not matched.

6. Match the applicants to the residency programs, and show intermediate steps of the algorithm. (5 points)
For this problem, suppose the applicant's preferences are given by:

Brady	Louis	Mary
City	City	City
General	General	General
Temple	Temple	Temple

Suppose that each residency program has only 1 open position, and that the program's preferences are given by:

General	City	Temple
Mary	Mary	Mary
Louis	Brady	Louis
Brady	Louis	Brady

Solution:

General	City	Temple
Louis	~~Brady~~	Brady
	Mary	