IEOR 151 – Midterm
October 21, 2015

Name:	
Overall:	/50

Instructions:

1. Show all your intermediate steps.
2. You are allowed a single 8.5x11 inch note sheet.
3. Calculators are allowed.
4. "Normal probability table" is given on last page.

1	/10
2	/10
3	/10
4	/20

1. Imagine that you are the manager of caffe strada and would like to determine the number of chocolate chip cookies that should be made in the morning using a newsvendor model with production costs.

   (a) Suppose demand $X \sim \mathcal{N}(\mu = 180, \sigma^2 = 120)$, selling price is \$1.80 (r = 1.8), the per unit production cost is \$1.20 ($c_v = 1.20$), and the holding cost is \$0.65 ($q = 0.65$). What is the optimal inventory level? (5 points)

   (b) Now, suppose the manager measures the demand for chocolate chip cookies for the past 20 days, and decides to use the nonparametric newsvendor model to solve the problem. The values of the demand, sorted into ascending order, are:
   103, 121, 142, 156, 164, 170, 173, 177, 179, 183, 184, 190, 192, 199, 200, 207, 215, 220, 231, 245.
   What is the optimal inventory level? (5 points)

Solutions:

(a) Given that X is normally distributed, we find δ^* such that $F(\delta^*) = \frac{r-c_v}{r+q} = \frac{1.8-1.2}{1.8+0.65} = 0.245$. From a z-table, we get that $\Phi(0.69) = 1 - 0.245 = 0.7550$. Hence, we would like to solve $\frac{\delta^*-180}{\sqrt{120}} = -0.69$. This yields $\delta^* = 173$ cookies.

(b) Consider the empirical CDF $\hat{F}(z) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}(z \leq X_i)$, and note that $\lceil n\frac{r-c_v}{r+q}\rceil = \lceil 20(\frac{1.8-1.2}{1.8+0.65})\rceil = \lceil 4.9 \rceil = 5$. Thus, you should purchase $X_5 = 164$ cookies.

2. Suppose you are the supply chain manager of Zara, and you would like to determine if the company should expand its production capacity by investing in a new factory in Vietnam. Analysts have reported that the demand for Zara goods have increased in the past year and forecasts continued growth. Investing in the new factory will cost \$520, and will allow Zara to increase production capacity by 10 units. If the current(future) average demand is 35, then investing in the new factory will yield \$650 in additional profits. However, if the current(future) average demand has remained unchanged as 27, the investment would be wasted. You have decided to use a minimax hypothesis testing approach to answer this question. As a first step, you record demand for goods over 10 days as follows:
22, 31, 31, 25, 23, 24, 26, 19, 35, 17.

   (a) Assume that the demand for Zara goods per day is approximated by a Gaussian random variable with variance $\sigma^2 = 90$. Using a binary search and z-table, compute the threshold for this hypothesis test γ^* to within an accuracy of ± 0.1 (4 points)
   Hint: Use the following values for the minimax hypothesis test: $n = 10$, $\mu_0 = 27$, $\mu_1 = 35$, $\sigma^2 = 90$, $L(\mu_0, d_0) = 0$, $L(\mu_0, d_1) = a = 520$, $L(\mu_1, d_0) = b = 650$, $L(\mu_1, d_1) = 0$. (8 points)

   (b) Should you invest in the new factory? Explain your answer? (2 points)

Solutions:

(a) Consider the following comparison, and recall that the goal is the select $\gamma*$ such that

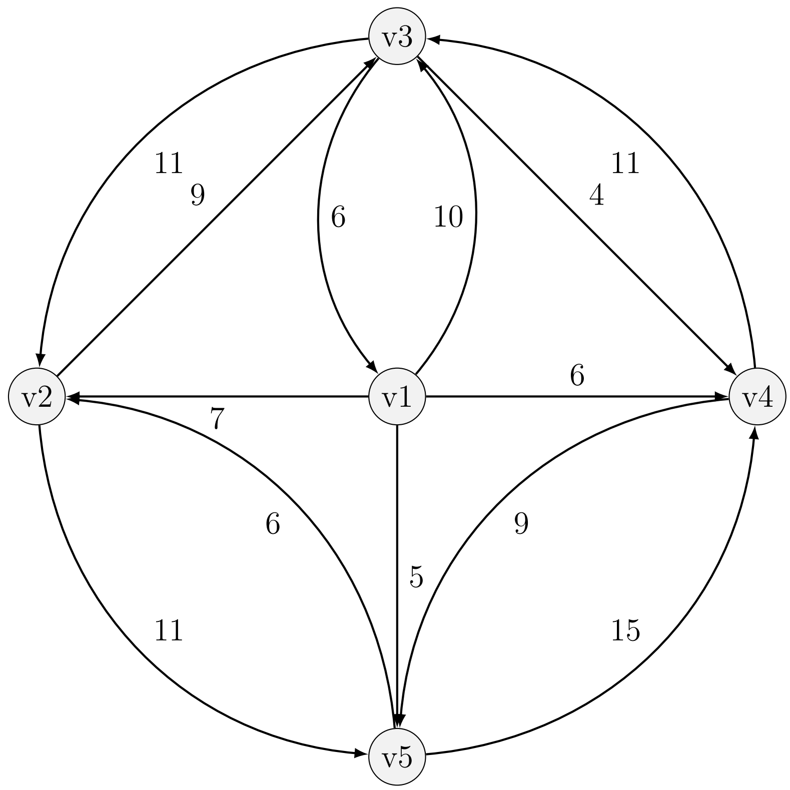
$$a(1 - \Phi(\frac{\sqrt{n}(\gamma^* - \mu_0)}{\sigma}) = b\Phi(\sqrt{n}\frac{(\gamma^* - \mu_1)}{\sigma})$$
$$520(1 - \Phi(\frac{\sqrt{10}(\gamma^* - 27)}{\sqrt{90}}) = 650\Phi(\sqrt{10}\frac{(\gamma^* - 35)}{\sqrt{90}}).$$

Since $a < b$, binary search should be conducted on $[27, 31]$ and the best first guess is 29. Note that the required accuracy concerns γ rather than the difference between LHS and RHS. Using binary search, we obtain $\gamma^* \in (30.8125, 30.875)$.

Step	γ	**LHS**	**RHS**
1	29	130.728	14.82
2	30	82.524	30.875
3	30.5	62.92	43.42
4	30.75	54.912	50.57
5	30.875	51.22	54.47
6	30.8125	53.04	52.52

(b) The manager decides d_0 and does not invest in the new factory as the sample mean 25.3 suggests that the demand has not increased (since $25.3 < \gamma^* = 30.81$).

3. Consider the following graph representation of a kidney exchange. Find the social welfare maximizing exchange under the constraint that all cycles can have length less than or equal to $L = 3$. (10 points)



Solutions:

First, list all cycles of length $L \leq 3$ and compute the weight of these cycles. Next, determine all sets of disjoint cycles and compute their weight. Lastly, the solution is the set of disjoint cycles with maximal weight. The steps are shown below, and the social welfare maximizing exchange are the disjoint cycles D and G.

Cycle Label	**Cycles of $L \leq 3$**	**Cycle Weight**	**Disjoint Cycles**	**Weight**
A	$v1 \to v3 \to v1$	16	F, E	40
B	$v2 \to v3 \to v2$	20	G, D	46
C	$v3 \to v4 \to v3$	15	B, D	44
D	$v4 \to v5 \to v4$	24	C, E	32
E	$v5 \to v2 \to v5$	17	A, E	33
F	$v1 \to v4 \to v3 \to v1$	23	A, D	40
G	$v1 \to v2 \to v3 \to v1$	22		

4. Consider the residency matching problem, and suppose the applicants' true preferences are given by:

Bob	**Linda**	**Tina**	**Gene**	**Louise**
1. General	1. City	1. City	1. City	1. City
2. City	2. Mercy	2. General	2. Mercy	2. General
		3. Mercy	3. General	3. Mercy

Additionally suppose that each residency program has 2 open positions, and that the true preferences of the programs are given by

Mercy	**City**	**General**
1. Tina	1. Bob	1. Gene
2. Louise	2. Linda	2. Tina
	3. Gene	3. Bob
	4. Tina	4. Linda
	5. Louise	

(a) Match the applicants to the residency programs, and show intermediate steps of the algorithm. (5 points)

(b) Now instead suppose that the order of the applicants and residency programs is switched in the matching algorithm. Match the applicants to the residency programs, and show intermediate steps of the algorithm. (5 points)

(c) Suppose City has hacked into the residency match system and knows everyone's true preferences. What is a false preference list that City can give that will lead to a better match for City when using the original algorithm? Show (by using the algorithm) why your choice leads to a better match for City. (10 points)

Solutions:

(a) The results are given by the following table.

Mercy	**City**	**General**
Louise	Linda	Bob
	~~Tina~~	Tina
	Gene	

(b) The results are given by the following table.

Bob	**Linda**	**Tina**	**Gene**	**Louise**
City	City	~~Mercy~~	General	Mercy
		General		

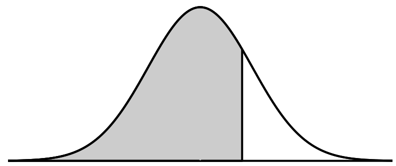
(c) Consider the preference list for residency programs, in which City has specified a false list of their preferences.

Mercy	**City**	**General**
1. Tina	1. Bob	1. Gene
2. Louise	2. Linda	2. Tina
		3. Bob
		4. Linda

In particular, City has truncated its preference list and has falsely deemed Gene, Tina, and Louise as unacceptable. This false preference list leads to an improved match for City when running the original algorithm. The resulting match from the original algorithm is given by the following table.

Mercy	**City**	**General**
Louise	Linda	~~Bob~~
	Bob	Tina
		Gene

Normal probability table



positive Z

Z	Second decimal place of Z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

*For $Z \geq 3.50$, the probability is greater than or equal to 0.9998.