IEOR 151 – Service Operations Design and Analysis

Homework 3 (Due 11/20/2015)

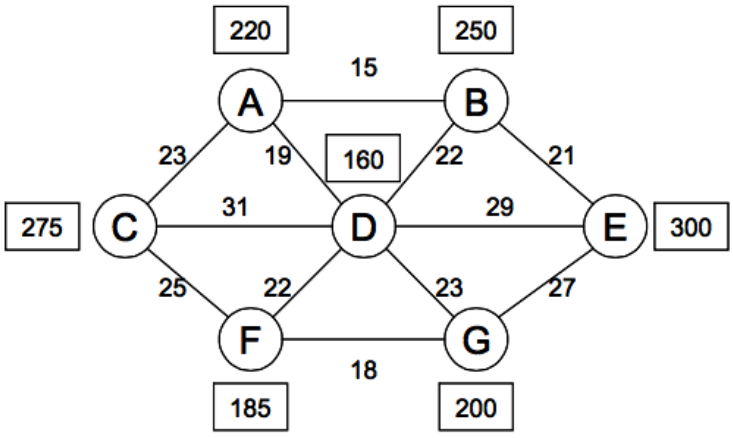
11/11/2015

1. In class, we looked at both the P-median and P-center model. The P-center model minimizes the maximum over all nodes distance between a demand node and the facility to which it is assigned, while the P-median model simply minimizes the demand weighted average distance. The constraints are the same for both problems. Therefore, we can use a **combined model** to find the tradeoff between the 2 objectives (minimizing the average and the maximum distances). Suppose we have two optimal solutions as shown in the table below.

Solution	**Avg. Distance**	**Max. Distance**
1	150	500
2	200	425

   - Given the 2 optimal solutions, what should be the value of W for the weighted objective - W(Avg. Dist) + (1-W)(Max. Dist)?
   - Sketch the tradeoff curve.

2. Practitioners are often interested in achieving multiple objectives with varying priorities (different from question 1).
   Consider the following network:
   Numbers shown in square beside the nodes are the demand per week at the node.
   Consider the hierarchical objective problem of **first** finding the minimum number of facilities needed to cover all nodes within a coverage distance of 23, and then, from among all of the alternate optima for this problem, **second** finding the solution that maximizes the number of demands that are covered two or more times.



   Define the following notation:
   Inputs:

(a) J: Set of demand nodes

(b) K: Set of candidate nodes (same as the set of demand nodes in this case)

(c) a_{jk}: 1 if demand node j is covered by a facility located at node k; 0 otherwise

(d) h_j: demand at node j

(e) W: weight on the primary objective of minimizing the number of facilities

Decision Variables:

(a) X_k: 1 if facility is located at candidate site k; 0 if not

(b) Z_j: 1 if demand node j is covered 2 or more times; 0 if not

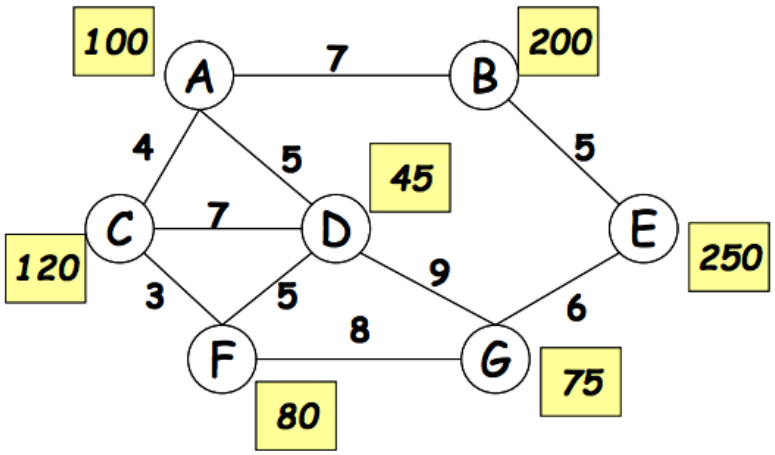
- Formulate the hierarchical objective of FIRST minimizing the number of facilities needed and SECOND maximizing the number of demands that are covered 2 or more times. Note that this should be a single function. This should be formulated in terms of notation.
- Formulate the constraints for this model including any integrality constraints. Explain the constraints using notations as well as words.
- Do the Z_j variables have to be constrained to be binary, or can you simply use a constraint of the following form: $0 \leq Z_j \leq 1 \forall j$
- How large should W be to ensure that the combined objective function of part (a) first minimizes the number of facilities and then selects the solution that maximizes the number of multiple covered demands from among the alternate optima? Briefly justify your answer. What you want is the smallest possible value of W.

3. Consider the nodes described below, and note that the depot is located at node 0. Suppose we would like to solve this vehicle routing problem (VRP) using the savings algorithm:

Distance	Node 0	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Node 8	Demand
Node 0		26	15	20	7	25	16	24	29	0
Node 1			15	23	26	33	40	38	54	18
Node 2				24	13	20	27	35	43	26
Node 3					26	42	34	15	39	11
Node 4						18	14	31	32	30
Node 5							25	49	45	21
Node 6								32	20	16
Node 7									30	29
Node 8										37

- Solve for the constraint that each vehicle has a capacity of 200
- Solve for the constraint that each vehicle has a capacity of 100
- Which solution yielded a better solution?

4. Solve a P-median algorithm with the heuristic algorithm: allocate 2 facilities among 7 demand nodes (demand nodes set and candidate sites set are the same). The demand and distance information is given in the figure below.



5. For the same graph in question 5, solve a set-covering problem with the heuristic algorithm: cover all demands with covering distance of 10.