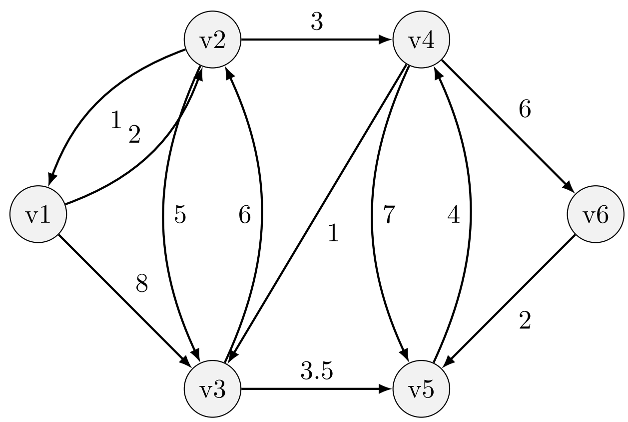
IEOR 151 – Service Operations Design and Analysis

Homework 2 (Due 10/16/2015)

October 14, 2015

1. Consider the following graph representation of a kidney exchange. Find the social welfare maximizaing exchange under the constraint that all cycles can have length less than or equal to L = 3. (5 points)

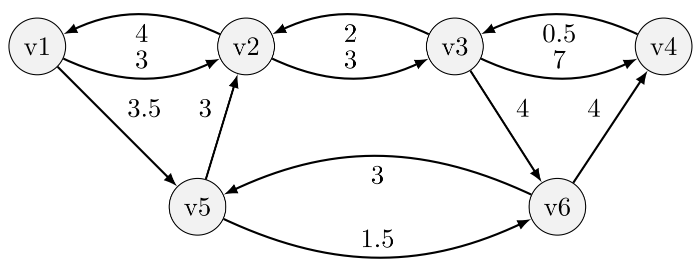


Solution:

Cycle Label	**Cycles of $L \leq 3$**	**Cycle Weight**	**Disjoint Cycles**	**Weight**
A	$v1 \to v2 \to v1$	3	A, G	15
B	$v2 \to v3 \to v2 \to v1$	11	A, F	11.5
C	$v4 \to v5 \to v4$	11	B, G	25
D	$v1 \to v3 \to v2 \to v1$	15	B, C	22
E	$v2 \to v4 \to v3 \to v2$	10	C, D	26
F	$v3 \to v5 \to v4 \to v3$	8.5	D, G	27
G	$v6 \to v5 \to v4 \to v6$	12	A, C	14

The Set of Disjoint Cycles {D, G} maximizes the social utility.

2. Consider the following graph representation of a kidney exchange. Find the social welfare maximizaing exchange under the constraint that all cycles can have length less than or equal to L = 3. (5 points)



Solution:

Cycle Label	**Cycles of $L \leq 3$**	**Cycle Weight**	**Disjoint Cycles**	**Weight**
A	$v1 \to v2 \to v1$	7	A, F	11.5
B	$v1 \to v5 \to v2 \to v1$	10.5	A, D	14.5
C	$v2 \to v3 \to v2$	5	A, F, D	19
D	$v3 \to v4 \to v3$	7.5	B, D	18
E	$v3 \to v6 \to v4 \to v3$	8.5	C, F	9.5
F	$v5 \to v6 \to v5 \to v3$	4.5	A, E	15.5
			B, E	19

Hence disjoint cycles {A, F, D} and {B, E} are optimal with weight 19.

3. Match the applicants to the residency programs, and show intermediate steps of the algorithm. (5 points)
For this problem, suppose the applicant's preferences are given by:

Manu	**Tony**	**Tim**	**Boris**
General	Hopkins	Temple	General
Hopkins	General	Hopkins	Temple
Temple	Temple	General	Hopkins

Suppose that each residency program has only 1 open position, and that the program's preferences are given by:

General	**Hopkins**	**Temple**
Boris	Manu	Boris
Manu	Tony	Manu
Tony	Tim	Tony
Tim	Boris	Tim

Solution:

General	**Hopkins**	**Temple**
~~Manu~~	~~Tony~~	~~Tim~~
Boris	Manu	Tony

Tim is unmatched.

4. Suppose Apple, Inc would like to purchase processors from Samsug Electronics Co. Apple's utility for the processors is given by $S(q) = 550\ln(1+q)$. The fixed costs for Samsung Electronics Co. are \$5,000, and if Samsung is inefficeint (efficient) then its marginal costs are 0.25 (0.20). Assume that Apple, Inc believes that there is a 40% chance that Samsung Electronics Co. is efficient.

   (a) What are the first-best production levels? (2 points)
   Solution:
   Note that equating marginal utility to marginal costs for the inefficient distributor gives:
   $S'(q_1^I) = \theta^I \Rightarrow q_1^I : \frac{500}{1+q_1^I} = \theta^I \Rightarrow q_1^I = 2,199$
   Similarly, equating marginal utility to marginal costs for the efficient distributor gives:
   $S'(q_1^E) = \theta^E \Rightarrow q_1^E : \frac{500}{1+q_1^E} = \theta^E \Rightarrow q_1^E = 2,749$

   (b) What are the contracts to implement the first-best production levels? (2 points)
   Solution:
   These contracts allow for zero information rent, meaning that the inefficient distributor should be offered the contract:
   $(q_1^I = 2,199, t_1^I = \theta_1^I q_1^I + F = 0.25 * 2199 + 5000 = 5,549.75)$

and the efficient distributor should be offered the contract:
$(q_1^E = 2,749, t_1^E = \theta_1^E q_1^E + F = 0.20 * 2749 + 5000 = 5,549.8)$

(c) How much profit would Samsung Electronics Co. make if Apple, Inc offers a menu of contracts $\{(q_1^I, t_1^I), (q_1^E, t_1^E)\}$ (1 point)
Solution:
If Samsung is inefficient, Profit = 0.
If Samsung is efficient, Profit $= t_1^I - \theta^E q_1^I - F = 5549.75 - (0.2)(2199) - 5000 = 109.95$

(d) What are the second-best production levels? (2 points)
Solution:
The production level for the efficient agent remains unchanged $q_2^E = q_1^E = 2,749$.
The production level for the inefficient agent decreases to:

$$q_2^I : S'(q_2^I) = \theta^I + \frac{\nu}{1-\nu}(\theta^I - \theta^E) \Rightarrow q_2^I : \frac{550}{1+q_2^I} = 0.25 + \frac{0.4}{0.6}(0.25 - 0.2) \Rightarrow q_2^I = 1942.5$$

(values close to 1940 are also accepted)

(e) What is the menu of contracts for the second-best production levels? (2 points)
Solution:
The transfer for the efficient agent is:
$t_2^E = \theta^E q_2^E + (\theta^I - \theta^E) q_2^I + F = (0.2)(2749) + (0.05)(1940) + 5000 = 5646.8$
and the transfer for the inefficient agent is:
$t_2^I = \theta^I q_2^I + F = (0.25)(1940) + 5000 = 5485$
Hence, the menu of contracts are $\{(q_2^E = 2,749, t_2^E = 5,646.8)(q_2^I = 1940, t_2^I = 5485)\}$

(f) What is the information rent of an efficient Samsung Electronics Co. for the menu of contracts for the second-best production levels? Is this higher or lower than the profit gained for the menu of contracts for the first-best production levels? (2 points)
Solution:
Information rent $= (\theta^I - \theta^E) q_2^1 = 0.05(1940) = 97 < 109.95$. Hence, Samsung gathers less profit/Information Rent with the second best contract.

5. Suppose a restaurant would like to purchase marinated steak from a meat distributor. The restaurant's utility for the steaks is given by $S(q) = 15\sqrt{q}$. The fixed costs for the distributors are \$35, and if the distributor is inefficeint (efficient) then its marginal costs are 0.35 (0.25). Assume that the restaurant believes that there is a 30% chance that the distributor is efficient.

(a) What are the first-best production levels? (2 points)
Solution:
Note that equating marginal utility to marginal costs for the inefficient distributor gives:
$S'(q_1^I) = \theta^I \Rightarrow q_1^I : \frac{15}{2\sqrt{q_1^I}} = 0.35 \Rightarrow q_1^I = 459.2(459 \text{ is accepted as well})$
Similarly, equating marginal utility to marginal costs for the efficient distributor gives:
$S'(q_1^E) = \theta^E \Rightarrow q_1^E : \frac{15}{2\sqrt{q_1^E}} = 0.30 \Rightarrow q_1^E = 900$

(b) What are the contracts to implement the first-best production levels? (2 points)
Solution:
These contracts allow for zero information rent, meaning that the inefficient distributor should be offered the contract:
$(q_1^I = 459, t_1^I = \theta_1^I q_1^I + F = (0.35)(459) + 35 = 195.65)$
and the efficient distributor should be offered the contract:
$(q_1^E = 900, t_1^E = \theta_1^E q_1^E + F = 0.25 * 900 + 35 = 260)$

(c) How much profit would the meat distributor make if the restaurant offers a menu of contracts $\{(q_1^I, t_1^I), (q_1^E, t_1^E)\}$ (1 point)
Solution:

If the meat distributor is inefficient, Profit $= 0$.
If the meat distributor is efficient, Profit $= t_1^I - \theta^E q_1^I - F = 195.65 - (0.25)(459) - 35 = 45.9$

(d) What are the second-best production levels? (2 points)
Solution:
The production level for the efficient agent remains unchanged $q_2^E = q_1^E = 900$.
The production level for the inefficient agent decreases to:

$$q_2^I : S'(q_2^I) = \theta^I + \frac{\nu}{1-\nu}(\theta^I - \theta^E) \Rightarrow q_2^I : \frac{15}{2\sqrt{q_2^I}} = 0.35 + \frac{0.3}{0.7}(0.1) \Rightarrow q_2^I = 364.4 (364 \text{ is accepted as well})$$

(e) What is the menu of contracts for the second-best production levels? (2 points)
Solution:
The transfer for the efficient agent is:
$t_2^E = \theta^E q_2^E + (\theta^I - \theta^E) q_2^I + F = (0.25)(900) + (0.1)(364) + 35 = 296.4$
and the transfer for the inefficient agent is:
$t_2^I = \theta^I q_2^I + F = ().35)(364_+ 35 = 162.4$
Hence, the menu of contracts are $\{(q_2^E = 900, t_2^E = 296.4(q_2^I = 364, t_2^I = 162.4)\}$

(f) What is the information rent of an efficient distributor for the menu of contracts for the second-best production levels? Is this higher or lower than the profit gained for the menu of contracts for the first-best production levels? (2 points)
Solution:
Information rent $= (\theta^I - \theta^E) q_2^1 = 0.1(364) = 36.4 < 45.9$. Hence, the meat distributor gathers less profit/Information Rent with the second best contract.