

# IEOR 151 – Service Operations Design and Analysis

## Homework 1 (Due 09/25/2015)

October 7, 2015

1. A newsboy would like to select the optimal level of inventory of newspapers using a newsvendor model without production costs. Here, demand  $X \sim \mathcal{U}(40, 200)$ , where  $\mathcal{U}$  is the uniform distribution, is in units of newspapers, holding costs are \$0.9 per newspaper, and the sale price is \$1.5 per newspaper. What is the optimal inventory level? (4 points)

Solution:

$$F(\delta^*) = \frac{p}{p+q} = \frac{1.5}{1.5+0.9} = 0.625$$

Given that  $X$  is uniformly distributed, we solve  $\delta^*$  as follows:  $\frac{\delta^* - 40}{160}$ . Hence,  $\delta^* = 140$  units

2. A real estate developer would like to select the optimal level of residential apartments to build in her new building using a newsvendor model with production costs. Here, demand  $X \sim \mathcal{N}(800, 200)$  is in apartment units, fixed costs are \$90,000,000, variable costs are \$750,000 per unit, holding costs are \$400,000 per apartment unit, and the (expected) sale price is \$1,200,000 per apartment unit. What is the optimal inventory level? (3 points)

Solution:

$$F(\delta^*) = \frac{p - c_v}{p + q} = \frac{1,200,000 - 750,000}{1,200,000 + 400,000} = 0.28125. \text{ Hence, } \delta^* = 800 - 0.58\sqrt{(200)} = 791.8$$

3. Suppose you are the manager of the Stuffed Inn and would like to determine the number of cups of clam chowder that should be made in the morning. Assume the selling price is \$2.25 ( $r = 2.25$ ), the per unit production cost is \$0.75 ( $c_v = 0.75$ ) and the holding cost is \$0.5 ( $q = 0.5$ ).

- Assume the demand is distributed as a Gaussian random variable with mean 500 and variance 1000. Use the newsvendor model with production cost to choose how many cups of soup to be produced. (4 points)

Solution:

$$F(\delta^*) = \frac{r - c_v}{r + q} = \frac{2.25 - 0.75}{2.25 + 0.5} = 0.5455. \text{ Hence, } z = 0.11 \text{ and } \delta^* = 500 + (0.114)\sqrt{1000} = 503.6$$

- Suppose you have measured the demand of soup (in cups) for the past 20 days, and the values of the demand, sorted into ascending order, are: 113, 197, 221, 234, 371, 375, 421, 469, 482, 495, 503, 559, 570, 611, 645, 672, 731, 757, 791, 834. Use the nonparametric newsvendor model to choose how many cups of soup the Stuffed Inn should produce. Explain your reasoning. (4 points)

Hint: Consider the sample distribution:

$$\hat{F}(z) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}(z \leq X_i)$$

Solution:

Considering the empirical CDF, we adopt the following approach -  $n \frac{r - c_v}{r + q} = 20 \frac{2.25 - 0.75}{2.25 + 0.5} = 10.9$ . Then the solution corresponds to the  $X_{\lceil n \frac{r - c_v}{r + q} \rceil} = X_{11}$ . Hence, we produce  $X_{11} = 503$  cups

4. Suppose you are the manager of the Port of Oakland, and you would like to determine if the port should purchase a new electronic system to manage the scheduling and routing of trucks (ground operations) of the port. Purchasing the system will cost \$5100, and other ports around the world have found that using the electronic system leads to an average of 19 mistakes (e.g. lost sales opportunities) per day. If the current error rate is 27 mistakes per day, then purchasing the new system will have a net savings

of \$4300 over the span of one year. You have decided to use a minimax hypothesis testing approach to answer this questions. As a first step, you record the number of mistakes made over 10 days: 29, 23, 22, 27, 28, 28, 31, 26, 21, 27.

Solution:

- Assume that the number of mistakes per day is approximated by a Gaussian random variable with variance  $\sigma^2 = 32.5$ . Using a binary search and  $z$ -table, compute the threshold for this hypothesis test  $\gamma^*$  to within an accuracy of  $\pm 0.1$  (4 points)

Hint: Use the following values for the minimax hypothesis test:  $n = 10$ ,  $\mu_0 = 19$ ,  $\mu_1 = 27$ ,  $\sigma^2 = 32.5$ ,  $L(\mu_0, \mu_0) = 0$ ,  $L(\mu_0, d_1) = a = 5100$ ,  $L(\mu_1, d_0) = b = 4300$ ,  $L(\mu_1, d_1) = 0$

Solution:

Consider the following comparison and recall that the goal is the select  $\gamma^*$  such that

$$a(1 - \Phi(\frac{\sqrt{n}(\gamma^* - \mu_0)}{\sigma})) = b\Phi(\frac{\sqrt{n}(\gamma^* - \mu_1)}{\sigma})$$

$$5100(1 - \Phi(\frac{\sqrt{10}(\gamma^* - 19)}{\sqrt{32.5}})) = 4300\Phi(\frac{\sqrt{10}(\gamma^* - 27)}{\sqrt{32.5}})$$

Since  $a > b$ , binary search should be conducted on [23, 27] and the best first guess is 25  
 Note, the required accuracy concerns gamma rather than the difference between LHS and RHS.  
 Using binary search, we obtain  $\gamma^* \in (23.0625, 23.125)$

Step	$\gamma$	LHS	RHS
1	25	2.23	574.6
2	24	14.14	206.6
3	23.5	32.02	112.24
4	23.25	46.92	80.66
5	23.125	56.43	67.94
6	23.0625	61.79	62.25

- Should the manager purchase the new electronic system? Explain your answer (2 points)

Solution:

$\bar{X}$  (Sample Mean) = 26.2  $>$   $\gamma^*$ .

Given the decision rule

$$\delta(X) = \begin{cases} d_0 & \text{if } \bar{X} \leq \gamma^* \\ d_1 & \text{if } \bar{X} > \gamma^* \end{cases}$$

The manager chooses  $d_1$  and buys the new system.

- Suppose you are managing the sales operation at an enterprise software company, and you would like to determine if the company should purchase a new electronic system to manage the sales process. Purchasing the system will cost \$6200, and other enterprise software companies have found that using the electronic system leads to an average of 3 mistakes (e.g. lost sales opportunities) per day. If the current error rate is 7 mistakes per day, then purchasing the new system will have a net savings of \$4500 over the span of one year. You have decided to use a minimax hypothesis testing approach to answer this questions. As a first step, you record the number of mistakes made over 10 days: 9, 3, 2, 7, 8, 8, 11, 6, 1, 7.

- Assume that the number of mistakes per day is approximated by a Gaussian random variable with variance  $\sigma^2 = 7.5$ . Using a binary search and  $z$ -table, compute the threshold for this hypothesis test  $\gamma^*$  to within an accuracy of  $\pm 0.1$  (4 points)

Hint: Use the following values for the minimax hypothesis test:  $n = 10$ ,  $\mu_0 = 3$ ,  $\mu_1 = 7$ ,  $\sigma^2 = 7.5$ ,  $L(\mu_0, \mu_0) = 0$ ,  $L(\mu_0, d_1) = a = 6200$ ,  $L(\mu_1, d_0) = b = 4500$ ,  $L(\mu_1, d_1) = 0$

Solution:

Consider the following comparison and recall that the goal is to select  $\gamma^*$  such that

$$a(1 - \Phi(\frac{\sqrt{n}(\gamma^* - \mu_0)}{\sigma})) = b\Phi(\frac{\sqrt{n}(\gamma^* - \mu_1)}{\sigma})$$

$$6200(1 - \Phi(\frac{\sqrt{10}(\gamma^* - 3)}{\sqrt{7.5}})) = 4300\Phi(\frac{\sqrt{10}(\gamma^* - 7)}{\sqrt{7.5}})$$

Since  $a > b$ , binary search should be conducted on  $[5, 7]$  and the best first guess is 6

Note, the required accuracy concerns gamma rather than the difference between LHS and RHS.

Using binary search, we obtain  $\gamma^* \in (5, 5.0625)$

Step	$\gamma$	LHS	RHS
1	6	1.65	558.48
2	5.5	12.07	187.3
3	5.25	29.06	97.44
4	5.125	43.83	68.36
5	5.0625	53.44	56.86

- Should the enterprise software company purchase the new electronic system? Explain your answer (2 points)

Solution:

$\bar{X}$  (Sample Mean) = 6.2 >  $\gamma^*$ .

Given the decision rule

$$\delta(X) = \begin{cases} d_0 & \text{if } \bar{X} \leq \gamma^* \\ d_1 & \text{if } \bar{X} > \gamma^* \end{cases}$$

The manager chooses  $d_1$  and buys the new system.