IEOR151

Lab 9
Solving P-median Problem in Excel

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Location Problems Summary

- Discrete Location Models
  - Covering
    - Set Covering
    - Maximal Covering
    - P-center
  - Average Distance
    - P-median
    - Fixed Charge

Another Classification

- Given number P facilities to locate
  - P-median
  - P-center
  - Maximal covering
- Decide number of facilities to locate
  - Set covering
  - Fixed-charge location problem

Maximal covering

- Maximize the total demand covered

$$\max \sum_{i \in I} h_i z_i$$

s.t.

$$\sum_{j \in J} x_j \leq p$$

$$z_i \leq \sum_{j \in J} a_{ij} x_j \;\; \forall i \in I$$

$$z_i \in \{0,1\}$$

$$x_j \in \{0,1\}$$

- $a_{ij} = 1$ if demand i can be served by facility j; 0 otherwise.
- $z_i = 1$ if demand i is covered
- $x_j = 1$ if facility location j is open

Fixed-charge Location Problem

- Minimize the total cost of facility investment and operating cost (e.g. transportation cost)
- Uncapacitated fixed-charge location problem (UFLP)

$$\min \sum_{j \in J} f_j x_j + \sum_{i \in I} \sum_{j \in J} h_i d_{ij} y_{ij}$$

s.t.

$$\sum_{j \in J} y_{ij} = 1, \forall i \in I$$

$$y_{ij} \le x_j, \forall i \in I, j \in J$$

$$x_j \in \{0,1\}$$

$$y_{ij} \in \{0,1\}$$

Heuristic Algorithms

- Not necessarily provide optimal solution
- Efficient way to find near-optimal solution(s)

P-median Problem Formulation

$$\min \sum_{j} \sum_{i} d_{ij} h_i y_{ij}$$

$st.$

$$\sum_{j} x_j = p$$

$$y_{ij} \le x_j, \forall i, j$$

$$\sum_{j} y_{ij} = 1, \forall i$$

$$x_j \in \{0,1\}$$

$$y_{ij} \in \{0,1\}$$

- h_i: demand at customer i
- D_{ij}: distance between customer i and site j
- P: number of facilities
- x_j: 1, if we locate at site j; 0, otherwise
- y_{ij}: 1, if customer i is served by site j; 0, otherwise

$$\sum_{i} y_{ij} \le I * x_j, \forall j \in J$$

$$0 \le y_{ij} \le 1$$

Example Layout

Allocating 2 facilities among 7 demand nodes (demand nodes set and candidate sites set are the same)

INPUTS

	A	B	C	D	E	F	G	Demand
A	0	7	4	5	12	7	14	100
B	7	0	11	12	5	14	11	200
C	4	11	0	7	16	3	11	120
D	5	12	7	0	15	5	9	45
E	12	5	16	15	0	14	6	250
F	7	14	3	5	14	0	8	80
G	14	11	11	9	6	8	0	75

Demand * Distance

	A	B	C	D	E	F	G
A	0	700	400	500	1200	700	1400
B	1400	0	2200	2400	1000	2800	2200
C	480	1320	0	840	1920	360	1320
D	225	540	315	0	675	225	405
E	3000	1250	4000	3750	0	3500	1500
F	560	1120	240	400	1120	0	640
G	1050	825	825	675	450	600	0

Number2Locate	2

Example Layout

DECISION VARIABLES

	A	B	C	D	E	F	G
Locations	0	0	1	0	1	0	0

Assignment Variables

	A	B	C	D	E	F	G
A	0	0	1	0	0	0	0
B	0	0	0	0	1	0	0
C	0	0	1	0	0	0	0
D	0	0	1	0	0	0	0
E	0	0	0	0	1	0	0
F	0	0	1	0	0	0	0
G	0	0	0	0	1	0	0

OBJECTIVE

Minimize	Demand weighted total	2405

Demand*Distance Matrix in "Input"

$$\min \sum_j \sum_i d_{ij} h_i y_{ij}$$

Example Layout

CONSTRAINTS

Node	Times Assigned		Required
A	1	=	1
B	1	=	1
C	1	=	1
D	1	=	1
E	1	=	1
F	1	=	1
G	1	=	1

$$\sum_j y_{ij} = 1, \forall i$$

$$\sum_j x_j = p$$

Sites		Allowed
2	<=	2

$$\sum_i y_{ij} \leq I * x_j$$

	Linkage Constraint (aggregate)						
	A	**B**	**C**	**D**	**E**	**F**	**G**
Nodes Assigned	0	0	4	0	3	0	0
	<=	<=	<=	<=	<=	<=	<=
Allowed	0	0	7	0	7	0	0

Don't forget that variables are binary!

P-median Problem Exercise

- Demand nodes: 1,2,3,4,5,6,7,8
- Candidate facility sites: A, B, C, D, E
- Distance information in table below
- Find 2 facility locations and optimal cost

	A	B	C	D	E	Demand
1	7	8	4	9	10	100
2	5	6	4	8	9	80
3	3	5	3	8	9	120
4	2	9	2	8	10	50
5	4	2	6	4	5	150
6	4	4	7	3	5	80
7	8	3	8	2	3	90
8	9	3	10	4	2	160