IEOR151 Homework 7 Solution

Fall 2013

Problem 1:

Solve a P-median problem with the heuristic algorithm: allocate 2 facilities among 7 demand nodes (demand nodes set and candidate sites set are the same). The demand and distance information is given in Fig.1:

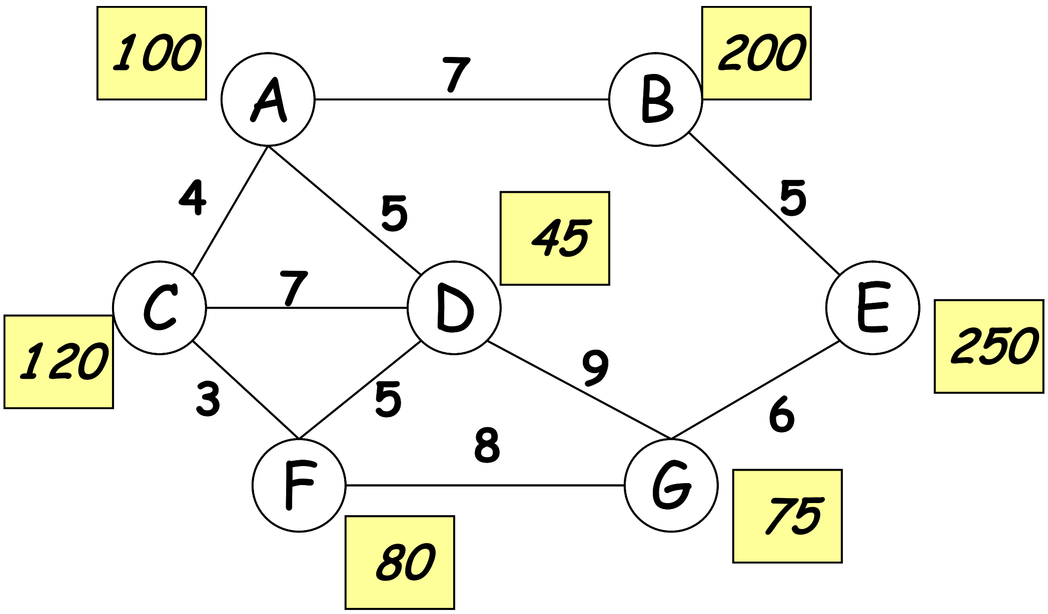

Fig.1

Solution: you may use Excel to compute the total travel distance.

1. Solve for 1-median problem. Locate 1st facility at B, with current total travel distance 5575.
2. Fixed 1st facility at B, compute total travel distance with 2nd facility opened at A, C, D,…,G. Locate 2nd facility at C. With B, C open, total travel distance is 3030.
3. Assign neighbors for B and C: {A,C,D,F,G}→ C and {E, B}→ B.
4. Solve 1-median problem in each neighbor
    a. In {A,C,D,F,G}, locate at C.
    b. In {E,B}, locate at E, since E has larger demand.
5. Check that there is no change in the neighbor. Stop the algorithm.
6. Solution is to locate at C and E with {A,C,D,F,G}→ C and {E, B}→ E.

Problem 2:

For the same graph Fig.1 in Problem 1, solve a set-covering problem with the heuristic algorithm: cover all demands with covering distance of 10.

Solution:

1. Let p=1, solve for p-center problem. The longest demand-facility distance assigned=14 > covering distance of 10. That means, some demand can not be covered with p=1.
2. Let p=2, solve for p-center problem. The longest demand-facility distance assigned=9 < covering distance of 10. That means, now all demands can be covered.
3. The minimum number of facilities needed to cover all demands is 2.

Problem 3 (Lab Assignment):

Please formulate the following service staffing problem as a LP problem and solve it with Excel Solver.

You are hired by a national drive-in bank to plan the workforce for their service operations. The planning horizon is 6 months from now on. They currently have 12 tellers. The workforce hours required for the next 6 months are 1500, 1800, 1600, 2000, 1800 and 2200. You have the option of hiring trainees at the beginning of each month. However, each trainees should be hired a month before they can start working since they need one month training period. The training requires 80 hours of workforce hour from a regular teller for each trainee. At the end of each month, 10 percent of regular workforce and trainees would quit. Each regular teller can work up to 160 hours each month including real work and education. The direct labor cost is $600 per month per teller, and $300 per month per trainee. (To clarify, a trainee would become a regular teller after one-month training period.)

For the purpose of this lab, we assume you can hire staff in fraction, i.e. some tellers might not work full month. Please formulate the LP problem first and then use Excel Solver to generate the optimal workforce plan in terms of number of trainees you hire each month and number of available tellers at each month.

Hint: You can use the following notations
Tt = number of trainees hired at the beginning of month t for t = 1,…6
At = number of tellers available at the beginning of month t for t = 1,…,6

Please turn in the LP formulation, printout of the Excel worksheet, the answer report from Excel Solver. You also need to summarize your workforce plan.

Solution:

It is OK to allow fractional assignments in this problem. The solver provides total cost =47354.

Model:

T_t = number of trainees hired at the beginning of month t for t = 1,…6

A_t = number of tellers available at the beginning of month t for t = 1,…,6

Minimize $\sum_{t=1}^{6} 600A_t + \sum_{t=1}^{6} 300T_t$

Such that: $A_1 = 12$

$$A_2 = 0.9(T_1 + A_1)$$

$$A_3 = 0.9(T_2 + A_2)$$

$$A_4 = 0.9(T_3 + A_3)$$

$$A_5 = 0.9(T_4 + A_4)$$

$$A_6 = 0.9(T_5 + A_5)$$

$$160A_1 - 80T_1 \ge 1500$$

$$160A_2 - 80T_2 \ge 1800$$

$$160A_3 - 80T_3 \ge 1600$$

$$160A_4 - 80T_4 \ge 2000$$

$$160A_5 - 80T_5 \ge 1800$$

$$160A_6 - 80T_6 \ge 2200$$

All variables non-negative

Note: The above formulation is assuming “trainees can quit”, if you assume “trainees can not quit”, replace constraints $A_i = 0.9(T_{i-1} + A_{i-1})$, i = 2,3,4,5,6 by $A_i = 0.9A_{i-1} + T_{i-1}$, i = 2,3,4,5,6